Chapter 1 • Introduction

P1.1 A gas at 20°C may be *rarefied* if it contains less than 10^{12} molecules per mm^3. If Avogadro's number is 6.023E23 molecules per mole, what air pressure does this represent?

Solution: The mass of one molecule of air may be computed as

$$m = \frac{\text{Molecular weight}}{\text{Avogadro's number}} = \frac{28.97\ \text{mol}^{-1}}{6.023\text{E}23\ \text{molecules/g}\cdot\text{mol}} = 4.81\text{E–}23\ \text{g}$$

Then the density of air containing 10^{12} molecules per mm^3 is, in SI units,

$$\rho = \left(10^{12}\ \frac{\text{molecules}}{\text{mm}^3}\right)\left(4.81\text{E–}23\ \frac{\text{g}}{\text{molecule}}\right)$$

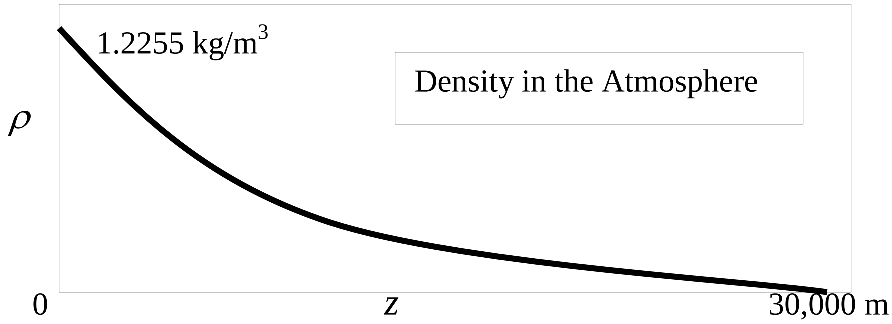
$$= 4.81\text{E–}11\ \frac{\text{g}}{\text{mm}^3} = 4.81\text{E–}5\ \frac{\text{kg}}{\text{m}^3}$$

Finally, from the perfect gas law, Eq. (1.13), at 20°C = 293 K, we obtain the pressure:

$$p = \rho RT = \left(4.81\text{E–}5\ \frac{\text{kg}}{\text{m}^3}\right)\left(287\ \frac{\text{m}^2}{\text{s}^2\cdot\text{K}}\right)(293\ \text{K}) = \mathbf{4.0\ Pa} \quad \textit{Ans.}$$

P1.2 Table A.6 lists the density of the standard atmosphere as a function of altitude. Use these values to estimate, crudely, say, within a factor of 2, the number of molecules of air in the entire atmosphere of the earth.

Solution: Make a plot of density ρ versus altitude z in the atmosphere, from Table A.6:



This writer's approximation: The curve is approximately an exponential, $\rho \approx \rho_o \exp(-b\, z)$, with b approximately equal to 0.00011 per meter. Integrate this over the entire atmosphere, with the radius of the earth equal to 6377 km:

$$m_{atmosphere} = \int \rho\, d(vol) \approx \int_0^{\infty} [\rho_o\, e^{-bz}](4\pi R_{earth}^2\, dz) =$$

$$= \frac{\rho_o\, 4\pi R_{earth}^2}{b} = \frac{(1.2255\, kg/m^3)4\pi(6.377E6\, m)^2}{0.00011/m} \approx 5.7E18\ kg$$

Dividing by the mass of one molecule ≈ 4.8E−23 g (see Prob. 1.1 above), we obtain the total number of molecules in the earth's atmosphere:

$$N_{molecules} = \frac{\text{m(atmosphere)}}{\text{m(one molecule)}} = \frac{\text{5.7E21 grams}}{\text{4.8E−23 gm/molecule}} \approx \mathbf{1.2E44}\ \ molecules \quad Ans.$$

This estimate, though crude, is within 10 per cent of the exact mass of the atmosphere.

P1.3 For the triangular element in Fig. P1.3, show that a tilted free liquid surface, in contact with an atmosphere at pressure p_a, must undergo shear stress and hence begin to flow.

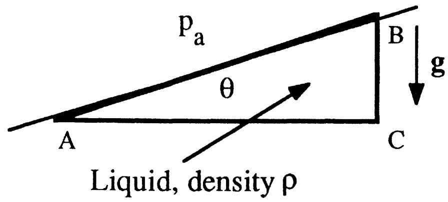
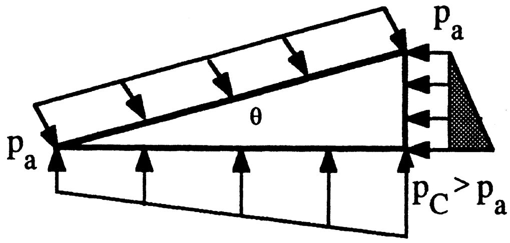


Fig. P1.3

Solution: Assume zero shear. Due to element weight, the pressure along the lower and right sides must vary linearly as shown, to a higher value at point C. Vertical forces are presumably in balance with element weight included. But horizontal forces are out of balance, with the unbalanced force being to the left, due to the shaded excess-pressure triangle on the right side BC. Thus hydrostatic pressures cannot keep the element in balance, and shear and flow result.



P1.4 Sand, and other granular materials, definitely *flow*, that is, you can pour them from a container or a hopper. There are whole textbooks on the "transport" of granular materials [54]. Therefore, is sand a *fluid*? Explain.

Solution: Granular materials do indeed *flow*, at a rate that can be measured by "flowmeters". But they are *not* true fluids, because they can support a small shear stress without flowing. They may rest at a finite angle without flowing, which is not possible for liquids (see Prob. P1.3). The maximum such angle, above which sand begins to flow, is called the *angle of repose*. A familiar example is sugar, which pours easily but forms a significant angle of repose on a heaping spoonful. The physics of granular materials are complicated by effects such as particle cohesion, clumping, vibration, and size segregation. See Ref. 54 to learn more.

P1.5 A formula for estimating the mean free path of a perfect gas is:

$$\ell = 1.26\frac{\mu}{\rho\sqrt{RT}} = 1.26\frac{\mu}{p}\sqrt{RT} \qquad (1)$$

where the latter form follows from the ideal-gas law, $\rho = p/RT$. What are the dimensions of the constant "1.26"? Estimate the mean free path of air at 20°C and 7 kPa. Is air *rarefied* at this condition?

Solution: We know the dimensions of every term except "1.26":

$$\{\ell\} = \{L\} \quad \{\mu\} = \left\{\frac{M}{LT}\right\} \quad \{\rho\} = \left\{\frac{M}{L^3}\right\} \quad \{R\} = \left\{\frac{L^2}{T^2\Theta}\right\} \quad \{T\} = \{\Theta\}$$

Therefore the above formula (first form) may be written dimensionally as

$$\{L\} = \{1.26?\}\frac{\{M/L\cdot T\}}{\{M/L^3\}\sqrt{[\{L^2/T^2\cdot\Theta\}\{\Theta\}]}} = \{1.26?\}\{L\}$$

Since we have {L} on both sides, {1.26} = {unity}, that is, the constant is dimensionless. The formula is therefore dimensionally homogeneous and should hold for any unit system.

For air at 20°C = 293 K and 7000 Pa, the density is $\rho = p/RT = (7000)/[(287)(293)] = 0.0832\ kg/m^3$. From Table A-2, its viscosity is 1.80E–5 $N\cdot s/m^2$. Then the formula predicts a mean free path of

$$\ell = 1.26\frac{1.80E{-}5}{(0.0832)[(287)(293)]^{1/2}} \approx \mathbf{9.4E{-}7\ m} \quad \textit{Ans.}$$

This is quite small. We would judge this gas to approximate a continuum if the physical scales in the flow are greater than about 100 ℓ, that is, greater than about 94 μm.

P1.6 Henri Darcy, a French engineer, proposed that the pressure drop Δp for flow at velocity V through a tube of length L could be correlated in the form

$$\frac{\Delta p}{\rho} = \alpha \; L V^2$$

If Darcy's formulation is consistent, what are the dimensions of the coefficient α?

Solution: From Table 1.2, introduce the dimensions of each variable:

$$\left\{\frac{\Delta p}{\rho}\right\} = \left\{\frac{ML^{-1}T^{-2}}{ML^{-3}}\right\} = \left\{\frac{L^2}{T^2}\right\} = \{\alpha L V^2\} = \{\alpha\}\{L\}\left\{\frac{L^2}{T^2}\right\}$$

Solve for $\{\alpha\} = \{L^{-1}\}$ *Ans.*

[The complete Darcy correlation is $\alpha = f/(2D)$, where D is the tube diameter, and f is a dimensionless friction factor (Chap. 6).]

P1.7 Convert the following inappropriate quantities into SI units: (*a*) 2.283E7 U.S. gallons per day; (*b*) 4.48 furlongs per minute (racehorse speed); and (*c*) 72,800 avoirdupois ounces per acre.

Solution: (*a*) (2.283E7 gal/day) x (0.0037854 m^3/gal) ÷ (86,400 s/day) = **1.0 m^3/s** *Ans.*(*a*)

(*b*) 1 furlong = (⅛)mile = 660 ft.

Then (4.48 furlongs/min)x(660 ft/furlong)x(0.3048 m/ft)÷(60 s/min) = **15 m/s** *Ans.*(*b*)

(*c*) (72,800 oz/acre)÷(16 oz/lbf)x(4.4482 N/lbf)÷(4046.9 acre/m^2) = **5.0 N/m^2 = 5.0 Pa** *Ans.*(*c*)

P1.8 Suppose that bending stress σ in a beam depends upon bending moment M and beam area moment of inertia I and is proportional to the beam half-thickness y. Suppose also that, for the particular case M = 2900 in·lbf, y = 1.5 in, and I = 0.4 in^4, the predicted stress is 75 MPa. Find the only possible dimensionally homogeneous formula for σ.

Solution: We are given that σ = y fcn(M,I) and we are *not* to study up on strength of materials but only to use dimensional reasoning. For homogeneity, the right hand side must have dimensions of stress, that is,

$$\{\sigma\} = \{y\}\{\text{fcn(M,I)}\}, \quad \text{or:} \quad \left\{\frac{M}{LT^2}\right\} = \{L\}\{\text{fcn(M,I)}\}$$

$$\text{or:} \quad \text{the function must have dimensions } \{\text{fcn(M,I)}\} = \left\{\frac{M}{L^2T^2}\right\}$$

Therefore, to achieve dimensional homogeneity, we somehow must combine bending moment, whose dimensions are $\{ML^2T^{-2}\}$, with area moment of inertia, $\{I\} = \{L^4\}$, and end up with $\{ML^{-2}T^{-2}\}$. Well, it is clear that {I} contains neither mass {M} nor time {T} dimensions, but the bending moment contains both mass and time and in exactly the combination we need, $\{MT^{-2}\}$. Thus it must be that σ *is proportional to M also*. Now we have reduced the problem to:

$$\sigma = yM\,\text{fcn(I)}, \quad \text{or} \quad \left\{\frac{M}{LT^2}\right\} = \{L\}\left\{\frac{ML^2}{T^2}\right\}\{\text{fcn(I)}\}, \quad \text{or:} \quad \{\text{fcn(I)}\} = \mathbf{\{L^{-4}\}}$$

We need just enough *I*'s to give dimensions of $\{L^{-4}\}$: we need the formula to be exactly *inverse* in *I*. The correct dimensionally homogeneous beam bending formula is thus:

$$\boldsymbol{\sigma} = \mathbf{C}\frac{\mathbf{My}}{\mathbf{I}}, \quad \text{where } \{C\} = \{\text{unity}\} \quad \textit{Ans.}$$

The formula admits to an arbitrary dimensionless constant C whose value can only be obtained from known data. Convert stress into English units: σ = (75 MPa)/(6894.8) = 10880 lbf/in^2. Substitute the given data into the proposed formula:

$$\sigma = 10880\ \frac{\text{lbf}}{\text{in}^2} = C\frac{My}{I} = C\frac{(2900\ \text{lbf·in})(1.5\ \text{in})}{0.4\ \text{in}^4}, \quad \text{or:} \quad \mathbf{C \approx 1.00} \quad \textit{Ans.}$$

The data show that C = 1, or $\boldsymbol{\sigma}$ **= My/I**, our old friend from strength of materials.

P1.9 A hemispherical container, 26 inches in diameter, is filled with a liquid at 20°C and weighed. The liquid weight is found to be 1617 ounces. (*a*) What is the density of the fluid, in kg/m^3? (*b*) What fluid might this be? Assume standard gravity, g = 9.807 m/s^2.

Solution: First find the volume of the liquid in m^3:

$$\text{Hemisphere volume} = \frac{1}{2}\left(\frac{\pi}{6}\right) D^3 = \frac{1}{2}\left(\frac{\pi}{6}\right)(26\ in)^3 = 4601\ in^3 \div \left(61024\frac{in^3}{m^3}\right)$$

$$= 0.0754 m^3$$

$$\text{Liquid mass} = 1617\ oz \div 16 = 101\ lbm\left(0.45359\frac{kg}{lbm}\right) = 45.84\ kg$$

$$\text{Then the liquid density} = \frac{45.84\ kg}{0.0754\ m^3} = \mathbf{607\frac{kg}{m^3}} \quad Ans.(a)$$

From Appendix Table A.3, this could very well be **ammonia**. *Ans*.(*b*)

P1.10 The Stokes-Oseen formula [10] for drag on a sphere at low velocity V is:

$$F = 3\pi\mu DV + \frac{9\pi}{16}\rho V^2 D^2$$

where D = sphere diameter, μ = viscosity, and ρ = density. Is the formula homogeneous?

Solution: Write this formula in dimensional form, using Table 1-2:

$$\{F\} = \{3\pi\}\{\mu\}\{D\}\{V\} + \left\{\frac{9\pi}{16}\right\}\{\rho\}\{V\}^2\{D\}^2 \ ?$$

$$\text{or:} \quad \left\{\frac{ML}{T^2}\right\} = \{1\}\left\{\frac{M}{LT}\right\}\{L\}\left\{\frac{L}{T}\right\} + \{1\}\left\{\frac{M}{L^3}\right\}\left\{\frac{L^2}{T^2}\right\}\{L^2\} \ ?$$

where, hoping for homogeneity, we have assumed that all constants (3,π,9,16) are *pure*, i.e., {unity}. Well, yes indeed, all terms have dimensions {ML/T^2}! Therefore the Stokes-Oseen formula (derived in fact from a theory) is ***dimensionally homogeneous.***

P1.11 In English Engineering units, the specific heat c_p of air at room temperature is approximately 0.24 Btu/(lbm-°F). When working with kinetic energy relations, it is more appropriate to express c_p as a velocity-squared per absolute degree. Give the numerical value, in this form, of c_p for air in (*a*) SI units, and (*b*) BG units.

Solution: From Appendix C, *Conversion Factors*, 1 Btu = 1055.056 J (or N-m) = 778.17 ft-lbf, and 1 lbm = 0.4536 kg = (1/32.174) slug. Thus the conversions are:

$$\text{SI units: } 0.24\frac{Btu}{lbm\,^\circ F} = 0.24\frac{1055.056\,N\cdot m}{(0.4536\,kg)(1K/1.8)} = 1005\frac{N\cdot m}{kg\cdot K} = \mathbf{1005}\frac{m^2}{s^2\,K} \quad Ans.(a)$$

$$\text{BG units: } 0.24\frac{Btu}{lbm\,^\circ F} = 0.24\frac{778.17\ ft\cdot lbf}{[(1/32.174)slug](1\,^\circ R)} = 6009\frac{ft\cdot lbf}{slug\cdot^\circ R} = \mathbf{6009}\frac{ft^2}{s^2\,^\circ R} \quad Ans.(b)$$

P1.12 For low-speed (laminar) flow in a tube of radius r_o, the velocity u takes the form

$$u = B\frac{\Delta p}{\mu}\left(r_o^2 - r^2\right)$$

where μ is viscosity and Δp the pressure drop. What are the dimensions of B?

Solution: Using Table 1-2, write this equation in dimensional form:

$$\{u\} = \{B\}\frac{\{\Delta p\}}{\{\mu\}}\{r^2\}, \quad \text{or:} \quad \left\{\frac{L}{T}\right\} = \{B?\}\frac{\{M/LT^2\}}{\{M/LT\}}\{L^2\} = \{B?\}\left\{\frac{L^2}{T}\right\},$$

$$\text{or:} \quad \mathbf{\{B\} = \{L^{-1}\}} \quad Ans.$$

The parameter B must have dimensions of inverse length. In fact, B is not a constant, it hides one of the variables in pipe flow. The proper form of the pipe flow relation is

$$u = C\frac{\Delta p}{L\mu}\left(r_o^2 - r^2\right)$$

where L is the *length of the pipe* and C is a dimensionless constant which has the theoretical laminar-flow value of (1/4)—see Sect. 6.4.

P1.13 The efficiency η of a pump is defined as

$$\eta = \frac{Q\Delta p}{\text{Input Power}}$$

where Q is volume flow and Δp the pressure rise produced by the pump. What is η if Δp = 35 psi, Q = 40 L/s, and the input power is 16 horsepower?

Solution: The student should perhaps verify that QΔp has units of power, so that η is a dimensionless ratio. Then convert everything to consistent units, for example, BG:

$$Q = 40\ \frac{L}{s} = 1.41\ \frac{ft^2}{s};\quad \Delta p = 35\ \frac{lbf}{in^2} = 5040\ \frac{lbf}{ft^2};\quad \text{Power} = 16(550) = 8800\ \frac{ft\cdot lbf}{s}$$

$$\eta = \frac{(1.41\ ft^3/s)(5040\ lbf/ft^2)}{8800\ ft\cdot lbf/s} \approx 0.81 \quad \text{or} \quad \mathbf{81\%} \quad \textit{Ans.}$$

Similarly, one could convert to SI units: Q = 0.04 m^3/s, Δp = 241300 Pa, and input power = 16(745.7) = 11930 W, thus h = (0.04)(241300)/(11930) = **0.81**. *Ans.*

P1.14 The volume flow Q over a dam is proportional to dam width B and also varies with gravity g and excess water height H upstream, as shown in Fig. P1.14. What is the only possible dimensionally homo-geneous relation for this flow rate?

Solution: So far we know that Q = B fcn(H,g). Write this in dimensional form:

$$\{Q\} = \left\{\frac{L^3}{T}\right\} = \{B\}\{f(H,g)\} = \{L\}\{f(H,g)\},$$

$$\text{or:}\quad \{f(H,g)\} = \left\{\frac{L^2}{T}\right\}$$

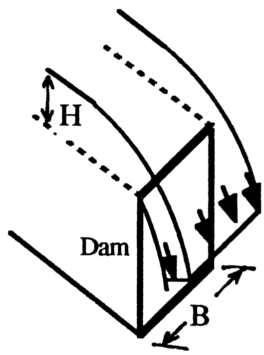


Fig. P1.14

So the function fcn(H,g) must provide dimensions of $\{L^2/T\}$, but only *g* contains *time*. Therefore g must enter in the form $g^{1/2}$ to accomplish this. The relation is now

$Q = Bg^{1/2}fcn(H)$, or: $\{L^3/T\} = \{L\}\{L^{1/2}/T\}\{fcn(H)\}$, or: $\{fcn(H)\} = \{\mathbf{L^{3/2}}\}$In order for fcn(H) to provide dimensions of $\{L^{3/2}\}$, the function must be a 3/2 power. Thus the final desired homogeneous relation for dam flow is:

$$\mathbf{Q = C\,B\,g^{1/2}\,H^{3/2},}\quad \text{where C is a dimensionless constant}\quad \textit{Ans.}$$

P1.15 The height *H* that fluid rises in a liquid barometer tube depends upon the liquid density *ρ*, the barometric pressure *p*, and the acceleration of gravity *g*. (a) Arrange these four variables into a single dimensionless group. (b) Can you deduce (or guess) the numerical value of your group?

Solution: This is a problem in *dimensional analysis*, covered in detail in Chapter 5. Use the symbols for dimensions suggested with Eq. (1.2): *M* for mass, *L* for length, *T* for time, *F* for force,

$$\{H\}= \{L\},\quad \{\rho\} = \{M/L^3\},\quad \{g\} = \{L/T^2\},\quad \{p\} = \{F/L^2\} = \{M/(LT^2)\}$$

where the change in pressure dimensions uses Newton's law, $\{F\} = \{ML/T^2\}$. We see that we can cancel mass by dividing density by pressure:

$$\left\{\frac{\rho}{p}\right\} = \left\{\frac{ML^{-3}}{ML^{-1}T^{-2}}\right\} = \left\{\frac{T^2}{L^2}\right\}$$

We can eliminate time by multiplying by $\{g\}$: $\{(\rho/p)(g)\} = \{(T^2/L^2)(L/T^2)\} = \{L^{-1}\}$. Finally, we can eliminate length by multiplying by the height $\{H\}$:

$$\left\{\left(\frac{\rho g}{p}\right)(H)\right\} = \{L^{-1}\}\{L\} = \{1\}\quad \textit{dimensionless}$$

Thus the desired dimensionless group is $\boldsymbol{\rho gH/p}$, or its inverse, $\boldsymbol{p/\rho gH}$. *Answer* (a)

(b) You might remember from physics, or other study, that the barometer formula is $p \approx \rho gH$. Thus this dimensionless group has a value of approximately 1.0, or **unity**. *Answer (*b*)*

P1.16 Test the dimensional homogeneity of the boundary-layer *x*-momentum equation:

$$\rho u\frac{\partial u}{\partial x}+\rho v\frac{\partial u}{\partial y}=-\frac{\partial p}{\partial x}+\rho g_x+\frac{\partial \tau}{\partial y}$$

Solution: This equation, like **all** theoretical partial differential equations in mechanics, is dimensionally homogeneous. Test each term in sequence:

$$\left\{\rho u\frac{\partial u}{\partial x}\right\}=\left\{\rho v\frac{\partial u}{\partial y}\right\}=\frac{M}{L^3}\frac{L}{T}\frac{L/T}{L}=\left\{\frac{\mathbf{M}}{\mathbf{L^2T^2}}\right\};\quad \left\{\frac{\partial p}{\partial x}\right\}=\frac{M/LT^2}{L}=\left\{\frac{\mathbf{M}}{\mathbf{L^2T^2}}\right\}$$

$$\{\rho g_x\}=\frac{M}{L^3}\frac{L}{T^2}=\left\{\frac{\mathbf{M}}{\mathbf{L^2T^2}}\right\};\quad \left\{\frac{\partial \tau}{\partial x}\right\}=\frac{M/LT^2}{L}=\left\{\frac{\mathbf{M}}{\mathbf{L^2T^2}}\right\}$$

All terms have dimension $\{ML^{-2}T^{-2}\}$. This equation may use *any* consistent units.

P1.17 Investigate the consistency of the Hazen-Williams formula from hydraulics:

$$Q=61.9D^{2.63}\left(\frac{\Delta p}{L}\right)^{0.54}$$

What are the dimensions of the constant "61.9"? Can this equation be used with confidence for a variety of liquids and gases?

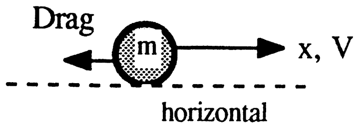
Solution: Write out the dimensions of each side of the equation:

$$\{Q\}=\left\{\frac{L^3}{T}\right\}\overset{?}{=}\{61.9\}\{D^{2.63}\}\left\{\frac{\Delta p}{L}\right\}^{0.54}=\{61.9\}\{L^{2.63}\}\left\{\frac{M/LT^2}{L}\right\}^{0.54}$$

The constant 61.9 has *fractional* dimensions: $\{61.9\} = \mathbf{\{L^{1.45}T^{0.08}M^{-0.54}\}}$ *Ans.*

Clearly, the formula is extremely inconsistent and cannot be used with confidence for any given fluid or condition or units. Actually, the Hazen-Williams formula, still in common use in the watersupply industry, is valid only for **water** flow in smooth pipes larger than 2-in. diameter and turbulent velocities less than 10 ft/s and (certain) English units. This formula should be held at arm's length and given a vote of "No Confidence."

***P1.18** ("*" means "difficult"—not just a plug-and-chug, that is) For small particles at low velocities, the first (linear) term in Stokes' drag law, Prob. 1.10, is dominant, hence F = KV, where K is a constant. Suppose a particle of mass m is constrained to move horizontally from the initial position x = 0 with initial velocity V = V_o. Show (a) that its velocity will decrease exponentially with time; and (b) that it will stop after travelling a distance x = mV_o/K.



Solution: Set up and solve the differential equation for forces in the *x*-direction:

$$\sum F_x = -\text{Drag} = ma_x, \quad \text{or:} \quad -KV = m\frac{dV}{dt}, \quad \text{integrate} \int_{V_o}^{V} \frac{dV}{V} = -\int_0^t \frac{m}{K}dt$$

$$\text{Solve} \quad \mathbf{V = V_o e^{-mt/K}} \quad \text{and} \quad \mathbf{x = \int_0^t V\,dt = \frac{mV_o}{K}\left(1 - e^{-mt/K}\right)} \quad \textit{Ans.}\,\text{(a,b)}$$

Thus, as asked, V drops off exponentially with time, and, as $t \to \infty, x = K\dfrac{V_o}{m}$

P1.19 In his study of the circular hydraulic jump formed by a faucet flowing into a sink, Watson [53] proposes a parameter combining volume flow rate *Q*, density ρ and viscosity μ of the fluid, and depth *h* of the water in the sink. He claims that the grouping is dimensionless, with *Q* in the numerator. Can you verify this?

Solution: Check the dimensions of these four variables, from Table 1.2:

$$\{Q\} = \{L^3/T\} \;;\; \{\rho\} = \{M/L^3\} \;;\; \{\mu\} = \{M/LT\} \;;\; \{h\} = \{L\}$$

Can we make this dimensionless? First eliminate mass {*M*} by dividing density by viscosity, that is, ρ/μ has units $\{T/L^2\}$. (I am pretending that kinematic viscosity is unfamiliar to the students in this introductory chapter.) Then combine ρ/μ and *Q* to eliminate time: $(\rho/\mu)Q$ has units {L}. Finally, divide that by a single depth *h* to form a dimensionless group:

$$\{\frac{\rho Q}{\mu h}\} = \frac{\{M/L^3\}\{L^3/T\}}{\{M/LT\}\{L\}} = \{1\} \quad = \text{dimensionless} \quad \textit{Ans.} \quad \text{Watson is correct.}$$

P1.20 Books on porous media and atomization claim that the viscosity μ and surface tension Υ of a fluid can be combined with a characteristic velocity U to form an important dimensionless parameter. (*a*) Verify that this is so. (*b*) Evaluate this parameter for water at 20°C and a velocity of 3.5 cm/s. NOTE: Extra credit if you know the name of this parameter.

Solution: We know from Table 1.2 that $\{\mu\} = \{ML^{-1}T^{-1}\}$, $\{U\} = \{LT^{-1}\}$, and $\{\Upsilon\} = \{FL^{-1}\} = \{MT^{-2}\}$. To eliminate mass {M}, we must divide μ by Υ, giving $\{\mu/\Upsilon\} = \{TL^{-1}\}$. Multiplying by the velocity will thus cancel all dimensions:

$$\frac{\mu U}{\Upsilon} \text{ is dimensionless, as is its inverse, } \frac{\Upsilon}{\mu U} \qquad \textit{Ans.(a)}$$

The grouping is called the *Capillary Number*. (*b*) For water at 20°C and a velocity of 3.5 cm/s, use Table A.3 to find μ = 0.001 kg/m-s and Υ = 0.0728 N/m. Evaluate

$$\frac{\mu U}{\Upsilon} = \frac{(0.001\,kg/m-s)(0.035\,m/s)}{(0.0728\,kg/s^2)} = \mathbf{0.00048}\,, \quad \frac{\Upsilon}{\mu U} = \mathbf{2080} \qquad \textit{Ans.(b)}$$

P1.21 Aeronautical engineers measure the pitching moment M_o of a wing and then write it in the following form for use in other cases:

$$M_o = \beta V^2 A C \rho$$

where V is the wing velocity, A the wing area, C the wing chord length, and ρ the air density. What are the dimensions of the coefficient β?

Solution: Write out the dimensions of each term in the formula:

$$\{M_o\} = \{FL\} = \left\{\frac{ML^2}{T^2}\right\} = \{\beta V^2 A C \rho\} = \{\beta\}\left\{\frac{L^2}{T^2}\right\}\{L^2\}\{L\}\left\{\frac{M}{L^3}\right\} = \left\{\frac{ML^2}{T^2}\right\}$$

Thus $\{\beta\}$ = {unity} or *dimensionless*. It is proportional to the *moment coefficient* in aerodynamics.

P1.22 The *Ekman number*, Ek, arises in geophysical fluid dynamics. It is a dimensionless parameter combining seawater density ρ, a characteristic length L, seawater viscosity μ, and the Coriolis frequency $\Omega \sin\phi$, where Ω is the rotation rate of the earth and ϕ is the latitude angle. Determine the correct form of Ek if the viscosity is in the numerator.

Solution : First list the dimensions of the various quantities:

$$\{\rho\} = \{ML^{-3}\} \; ; \quad \{L\} = \{L\} \; ; \quad \{\mu\} = \{ML^{-1}T^{-1}\} \; ; \quad \{\Omega \sin\phi\} = \{T^{-1}\}$$

Note that $\sin\phi$ is itself dimensionless, so the Coriolis frequency has the dimensions of Ω. Only ρ and μ contain mass {M}, so if μ is in the numerator, ρ must be in the denominator. That combination μ/ρ we know to be the kinematic viscosity, with units $\{L^2T^{-1}\}$. Of the two remaining variables, only $\Omega \sin\phi$ contains time $\{T^{-1}\}$, so it must be in the denominator. So far, we have the grouping $\mu/(\rho\,\Omega \sin\phi)$, which has the dimensions $\{L^2\}$. So we put the length-squared into the denominator and we are finished:

$$\text{Dimensionless Ekman number:} \quad \text{Ek} = \frac{\mu}{\rho L^2 \Omega \sin\phi} \qquad \textit{Ans.}$$

P1.23 During World War II, Sir Geoffrey Taylor, a British fluid dynamicist, used dimensional analysis to estimate the energy released by an atomic bomb explosion. He assumed that the energy released, E, was a function of blast wave radius R, air density ρ, and time t. Arrange these variables into a single dimensionless group, which we may term the *blast wave number*.

Solution: These variables have the dimensions $\{E\} = \{ML^2/T^2\}$, $\{R\} = \{L\}$, $\{\rho\} = \{M/L^3\}$, and $\{t\} = \{T\}$. Multiplying E by t^2 eliminates time, then dividing by ρ eliminates mass, leaving $\{L^5\}$ in the numerator. It becomes dimensionless when we divide by R^5. Thus

$$\text{Blast wave number} = \frac{E\,t^2}{\rho R^5}$$

P1.24 Air, assumed to be an ideal gas with $k = 1.40$, flows isentropically through a nozzle. At section 1, conditions are sea level standard (see Table A.6). At section 2, the temperature is –50°C. Estimate (*a*) the pressure, and (*b*) the density of the air at section 2.

Solution: From Table A.6, $p_1 = 101350$ Pa, $T_1 = 288.16$ K, and $\rho_1 = 1.2255$ kg/m^3. Convert to absolute temperature, $T_2 = -50°C = 223.26$ K. Then, for a perfect gas with constant k,

$$\frac{p_2}{p_1} = \left(\frac{T_2}{T_1}\right)^{k/(k-1)} = \left(\frac{223.16}{288.16}\right)^{1.4/(1.4-1)} = (0.7744)^{3.5} = 0.4087$$

$$\text{Thus} \quad p_2 = (0.4087)(101350\,Pa) = \mathbf{41,400}\ Pa \quad Ans.(a)$$

$$\frac{\rho_2}{\rho_1} = \left(\frac{T_2}{T_1}\right)^{1/(k-1)} = \left(\frac{223.16}{288.16}\right)^{1/(1.4-1)} = (0.7744)^{2.5} = 0.5278$$

$$\text{Thus} \quad \rho_2 = (0.5278)(1.2255 kg/m^3) = \mathbf{0.647}\ kg/m^3 \quad Ans.(b)$$

Alternately, once p_2 was known, we could have simply computed ρ_2 from the ideal-gas law.

$$\rho_2 = p_2/RT_2 = (41400)/[287(223.16)] = 0.647 \text{ kg/m}^3$$

P1.25 On a summer day in Narragansett, Rhode Island, the air temperature is 74ºF and the barometric pressure is 14.5 lbf/in^2. Estimate the air density in kg/m^3.

Solution: This is a problem in handling awkward units. Even if we use the BG system, we have to convert. But, since the problem calls for a metric result, better we should convert to SI units:

$$T = 74ºF + 460 = 534ºR \times 0.5556 \ \text{(inside front cover)} = 297 \text{ K}$$

$$p = 14.5 \text{ lbf/in}^2 \times 6894.8 \ \text{(inside front cover)} = 100{,}700 \text{ Pa}$$

The SI gas constant, from Eq. (1.12), is 287 m^2/(s^2·K). Thus, from the ideal gas law, Eq. (1.10),

$$\rho = \frac{p}{RT} = \frac{100{,}700 \text{ N/m}^2}{(287 \frac{\text{m}^2}{\text{s}^2 \cdot \text{K}})(297 \text{ K})} = 1.18 \frac{\text{N} \cdot \text{s}^2}{\text{m}^4}$$

This doesn't look like a density unit, until we realize that 1 N ≡ 1 kg·m/s^2. Making this substitution, we find that

$$\rho = \mathbf{1.18\ kg/m^3} \qquad \textit{Answer}$$

Notice that faithful use of SI units will lead to faithful SI results, without further conversion.

P1.26 A tire has a volume of 3.0 ft^3 and a 'gage' pressure (above atmospheric pressure) of 32 psi at 75°F. If the ambient pressure is sea-level standard, what is the weight of air in the tire?

Solution: Convert the temperature from 75°F to 535°R. Convert the pressure to psf:

$$p = (32\ \text{lbf/in}^2)(144\ \text{in}^2/\text{ft}^2) + 2116\ \text{lbf/ft}^2 = 4608 + 2116 \approx 6724\ \text{lbf/ft}^2$$

From this compute the density of the air in the tire:

$$\rho_{air} = \frac{p}{RT} = \frac{6724\ \text{lbf/ft}^2}{(1717\ \text{ft·lbf/slug·°R})(535\text{°R})} = 0.00732\ \text{slug/ft}^3$$

Then the total weight of air in the tire is

$$W_{air} = \rho g \upsilon = (0.00732\ \text{slug/ft}^3)(32.2\ \text{ft/s}^2)(3.0\ \text{ft}^3) \approx \mathbf{0.707\ lbf} \quad \textit{Ans.}$$

P1.27 For steam at a pressure of 45 atm, some values of temperature and specific volume are as follows, from Ref. 23:

T, ºF	500	600	700	800	900
v, ft^3/lbm	0.7014	0.8464	0.9653	1.074	1.177

Find an average value of the predicted gas constant R in $m^2/(s^2\cdot K)$. Does this data reasonably approximate an ideal gas? If not, explain.

Solution: If ideal, the calculated gas constant would, from Table A.4, be about 461 $m^2/(s^2\cdot K)$. Try this for the first value, at T = 500ºF = 960ºR. Change to SI units, using the inside front cover conversions:

$$T = 500\text{ºF} = 960\text{ºR} \div 1.8 = 533\ \text{K}; \quad p = 45(101{,}350\ \text{Pa}) = 4561\ \text{kPa}$$

$$v = 0.7014\ \text{ft}^3/\text{lbm} \div 16.019 = 0.0438\ \text{m}^3/\text{kg}$$

$$p\,v = R\,T\text{, or: } R = \frac{p\,v}{T} = \frac{(4.561E6\ Pa)(0.0438\ m^3/kg)}{533\ K} = 374\ \frac{m^2}{s^2\cdot K}$$

This is about 19% lower than the recommended value of R_{steam} = 461 $m^2/(s^2\cdot K)$. Continue by filling out the rest of the table:

T, ºF	500	600	700	800	900
R, $m^2/(s^2\cdot K)$	374	409	427	437	443

These are *all* low, with an average of 418, nine per cent low. The temperature is too low and the pressure too high. We are too near the saturation line for the ideal gas law to be accurate.

P1.28 Wet air, at 100% relative humidity, is at 40°C and 1 atm. Using Dalton's law of partial pressures, compute the density of this wet air and compare with dry air.

Solution: Change T from 40°C to 313 K. Dalton's law of partial pressures is

$$p_{tot} = 1\text{ atm} = p_{air} + p_{water} = \frac{m_a}{\upsilon}R_aT + \frac{m_w}{\upsilon}R_wT$$

$$\text{or:}\quad m_{tot} = m_a + m_w = \frac{p_a\upsilon}{R_aT} + \frac{p_w\upsilon}{R_wT}\quad \text{for an ideal gas}$$

where, from Table A-4, $R_{air} = 287$ and $R_{water} = 461\ m^2/(s^2\cdot K)$. Meanwhile, from Table A-5, at 40°C, the vapor pressure of saturated (100% humid) water is 7375 Pa, whence the partial pressure of the air is $p_a = 1\text{ atm} - p_w = 101350 - 7375 = 93975$ Pa.

Solving for the mixture density, we obtain

$$\rho = \frac{m_a + m_w}{\upsilon} = \frac{p_a}{R_aT} + \frac{p_w}{R_wT} = \frac{93975}{287(313)} + \frac{7375}{461(313)} = 1.046 + 0.051 \approx \mathbf{1.10\ \frac{kg}{m^3}}\quad \textit{Ans.}$$

By comparison, the density of dry air for the same conditions is

$$\rho_{dry\ air} = \frac{p}{RT} = \frac{101350}{287(313)} = 1.13\ \frac{kg}{m^3}$$

Thus, at 40°C, wet, 100% humidity, air is *lighter* than dry air, by about **2.7%**.

P1.29 A tank holds 5 ft^3 of air at 20°C and 120 psi (gage). Estimate the energy in ft-lbf required to compress this air isothermally from one atmosphere (14.7 psia = 2116 psfa).

Solution: Integrate the work of compression, assuming an ideal gas:

$$W_{1\text{-}2} = -\int_1^2 p\,d\upsilon = -\int_1^2 \frac{mRT}{\upsilon}d\upsilon = -mRT\ln\left(\frac{\upsilon_2}{\upsilon_1}\right) = p_2\upsilon_2\ln\left(\frac{p_2}{p_1}\right)$$

where the latter form follows from the ideal gas law for isothermal changes. For the given numerical data, we obtain the quantitative work done:

$$W_{1\text{-}2} = p_2\upsilon_2\ \ln\left(\frac{p_2}{p_1}\right) = \left(134.7\times 144\ \frac{lbf}{ft^2}\right)(5\ ft^3)\ \ln\left(\frac{134.7}{14.7}\right) \approx \mathbf{215{,}000\ ft\cdot lbf}\quad \textit{Ans.}$$

P1.30 Repeat Prob. 1.29 if the tank is filled with compressed *water* rather than air. Why is the result thousands of times less than the result of 215,000 ft·lbf in Prob. 1.29?

Solution: First evaluate the density change of water. At 1 atm, $\rho_o \approx 1.94$ slug/ft^3. At 120 psi(gage) = 134.7 psia, the density would rise slightly according to Eq. (1.22):

$$\frac{p}{p_o} = \frac{134.7}{14.7} \approx 3001\left(\frac{\rho}{1.94}\right)^7 - 3000, \quad \text{solve } \rho \approx 1.940753 \text{ slug/ft}^3,$$

$$\text{Hence } m_{water} = \rho\upsilon = (1.940753)(5 \text{ ft}^3) \approx 9.704 \text{ slug}$$

The density change is extremely small. Now the work done, as in Prob. 1.29 above, is

$$W_{1\text{-}2} = -\int_1^2 p\,d\upsilon = \int_1^2 p\,d\left(\frac{m}{\rho}\right) = \int_1^2 p\,\frac{m\,d\rho}{\rho^2} \approx p_{avg} m \frac{\Delta\rho}{\rho_{avg}^2} \quad \text{for a linear pressure rise}$$

$$\text{Hence } W_{1\text{-}2} \approx \left(\frac{14.7+134.7}{2}\times 144\ \frac{\text{lbf}}{\text{ft}^2}\right)(9.704\text{ slug})\left(\frac{0.000753}{1.9404^2}\ \frac{\text{ft}^3}{\text{slug}}\right) \approx \mathbf{21\ ft{\cdot}lbf} \quad \textit{Ans.}$$

[Exact integration of Eq. (1.22) would give the same numerical result.] Compressing water (extremely small $\Delta\rho$) takes *ten thousand times less energy* than compressing air, which is why it is safe to test high-pressure systems with water but dangerous with air.

P1.31 One cubic foot of argon gas at 10°C and 1 atm is compressed isentropically to a new pressure of 600 kPa. (*a*) What will be its new density and temperature? (*b*) If allowed to cool, at this new volume, back to 10°C, what will be the final pressure? Assume constant specific heats.

Solution: This is an exercise in having students recall their thermodynamics. From Table A.4, for argon gas, $R = 208$ m^2/(s^2-K) and $k = 1.67$. Note $T_1 = 283K$. First compute the initial density:

$$\rho_1 = \frac{p_1}{RT_1} = \frac{101350\,N/m^2}{(208\,m^2/s^2\cdot K)(283K)} = 1.72\ kg/m^3$$

For an isentropic process at constant *k*,

$$\frac{p_2}{p_1} = \frac{600,000\,Pa}{101,350\,Pa} = 5.92 = \left(\frac{\rho_2}{\rho_1}\right)^k = \left(\frac{\rho_2}{1.72}\right)^{1.67}, \text{ Solve } \rho_2 = \mathbf{4.99\ kg/m^3} \quad \textit{Ans.}(a)$$

$$\frac{p_2}{p_1} = 5.92 = \left(\frac{T_2}{T_1}\right)^{k/(k-1)} = \left(\frac{T_2}{283K}\right)^{1.67/0.67}, \text{ Solve } T_2 = \mathbf{578\,K} = 305^\circ C \quad \textit{Ans.}(a)$$

(*b*) Cooling at constant volume means ρ stays the same and the new temperature is

$$p_3 = \rho_3 R T_3 = (5.00\frac{kg}{m^3})(208\frac{m^2}{s^2K})(283K) = 294{,}000\,Pa = \mathbf{294\ kPa} \quad \textit{Ans.}(b)$$

283K. Thus

P1.32 A blimp is approximated by a prolate spheroid 90 m long and 30 m in diameter. Estimate the weight of 20°C gas within the blimp for (a) helium at 1.1 atm; and (b) air at 1.0 atm. What might the difference between these two values represent (Chap. 2)?

Solution: Find a handbook. The volume of a prolate spheroid is, for our data,

$$\upsilon = \frac{2}{3}\pi LR^2 = \frac{2}{3}\pi(90\text{ m})(15\text{ m})^2 \approx 42412\text{ m}^3$$

Estimate, from the ideal-gas law, the respective densities of helium and air:

$$\text{(a)}\quad \rho_{helium} = \frac{p_{He}}{R_{He}T} = \frac{1.1(101350)}{2077(293)} \approx 0.1832\ \frac{kg}{m^3};$$

$$\text{(b)}\quad \rho_{air} = \frac{p_{air}}{R_{air}T} = \frac{101350}{287(293)} \approx 1.205\ \frac{kg}{m^3}.$$

Then the respective gas weights are

$$W_{He} = \rho_{He} g \upsilon = \left(0.1832\ \frac{kg}{m^3}\right)\left(9.81\ \frac{m}{s^2}\right)(42412\text{ m}^3) \approx \mathbf{76000\ N} \quad \textit{Ans. (a)}$$

$$W_{air} = \rho_{air} g \upsilon = (1.205)(9.81)(42412) \approx \mathbf{501000\ N} \quad \textit{Ans. (b)}$$

The difference between these two, **425000 N**, is the *buoyancy*, or lifting ability, of the blimp. [See Section 2.8 for the principles of buoyancy.]

P1.33 A tank contains 9 kg of CO_2 at 20ºC and 2.0 MPa. Estimate the volume of the tank, in m^3.

Solution: All we have to do is find the density. For CO_2, from Table A.4, R = 189 $m^2/(s^2\cdot K)$. Then

$$\rho = \frac{p}{R\,T} = \frac{(2{,}000{,}000\ Pa)}{([189\ \text{m}^2/(\text{s}^2\cdot\text{K}](20+273\ \text{K})} = 36.1\ kg/m^3$$

Then the tank volume $= m/\rho = (9\,kg)/(36.1\,kg/m^3) = 0.25\,m^3$ *Ans.*

P1.34 Consider steam at the following state near the saturation line: $(p_1, T_1) = (1.31$ MPa, 290°C). Calculate and compare, for an ideal gas (Table A.4) and the Steam Tables (a) the density ρ_1; and (b) the density ρ_2 if the steam expands isentropically to a new pressure of 414 kPa. Discuss your results.

Solution: From Table A.4, for steam, $k \approx 1.33$, and $R \approx 461\ m^2/(s^2\cdot K)$. Convert T_1 = 563 K. Then,

$$\rho_1 = \frac{p_1}{RT_1} = \frac{1{,}310{,}000\ Pa}{(461\ m^2/s^2K)(563\ K)} = \mathbf{5.05}\ \frac{kg}{m^3} \quad Ans.\ (a)$$

$$\frac{\rho_2}{\rho_1} = \frac{\rho_2}{5.05} = \left(\frac{p_2}{p_1}\right)^{1/k} = \left(\frac{414\ kPa}{1310\ kPa}\right)^{1/1.33} = 0.421, \quad or: \quad \rho_2 = \mathbf{2.12}\ \frac{kg}{m^3} \quad Ans.\ (b)$$

From the online Steam Tables, just look it up:

$$SpiraxSarco\,Tables: \quad \rho_1 = \mathbf{5.23}\ \text{kg/m}^3 \quad Ans.\ (a), \qquad \rho_2 = \mathbf{2.16}\ \text{kg/m}^3 \quad Ans.\ (b)$$

The ideal-gas error is only about 3%, even though the expansion approached the saturation line.

P1.35 In Table A-4, most common gases (air, nitrogen, oxygen, hydrogen, CO, NO) have a specific heat ratio $k = 1.40$. Why do argon and helium have such high values? Why does NH_3 have such a low value? What is the lowest k for any gas that you know?

Solution: In elementary kinetic theory of gases [21], k is related to the number of “degrees of freedom” of the gas: $k \approx 1 + 2/N$, where N is the number of different modes of translation, rotation, and vibration possible for the gas molecule.

Example: Monotomic gas, N = 3 (translation only), thus $k \approx 5/3$

This explains why helium and argon, which are monatomic gases, have $k \approx 1.67$.

Example: Diatomic gas, N = 5 (translation plus 2 rotations), thus $k \approx 7/5$

This explains why air, nitrogen, oxygen, NO, CO and hydrogen have $k \approx 1.40$.

But NH_3 has *four* atoms and therefore more than 5 degrees of freedom, hence k will be less than 1.40. Most tables list $k = 1.32$ for NH_3 at about 20°C, implying N ≈ 6.

The lowest k known to this writer is for *uranium hexafluoride*, $^{238}UF_6$, which is a very complex, heavy molecule with many degrees of freedom. The estimated value of k for this heavy gas is $k \approx$ **1.06**.

P1.36 Experimental data [55] for the density of n-pentane liquid for high pressures, at 50ºC, are listed as follows:

Pressure, kPa	100	10230	20700	34310
Density, kg/m^3	586.3	604.1	617.8	632.8

(*a*) Fit this data to reasonably accurate values of *B* and *n* from Eq. (1.19).
(*b*) Evaluate ρ at 30 MPa.

Solution:
Eq. (1.19) is $p/p_o \approx (B+1)(\rho/\rho_o)^n - B$. The first column is p_o = 100 kPa and ρ_o = 586.3 kg/m^3.

(*a*) The writer found it easiest to guess *n*, say, *n* = 7 for water, and then solve for *B* from the data:

$$B = \frac{[\frac{p}{p_o} - \left(\frac{\rho}{\rho_o}\right)^n]}{(\frac{\rho}{\rho_o})^n - 1}$$

The value *n* = 7 is too low. Try *n* = 8,9,10,11 – yes, 11 works, with $B \approx 260 \pm 2$.
Equation (1.19), with ***B* = 260 and *n* = 11**, is accurate to a fraction of 1%. *Ans*.(*a*)

(*b*) Use our new formula for *p* = 30 MPA = 30,000 kPa.

$$\frac{p}{p_o} = \frac{30000}{100} = (261)\left(\frac{\rho}{586.1}\right)^{11} - 260\,, \qquad solve \;\; \rho \approx \mathbf{628}\ \frac{\boldsymbol{kg}}{\boldsymbol{m^3}} \quad Ans.(b)$$

P1.37 A near-ideal gas has M = 44 and c_V = 610 J/(kg·K). At 100°C, what are (a) its specific heat ratio, and (b) its speed of sound?

Solution: The gas constant is $R = \Lambda/M = 8314/44 \approx 189$ J/(kg·K). Then

$$c_v = R/(k-1), \quad \text{or:} \quad k = 1 + R/c_v = 1 + 189/610 \approx \mathbf{1.31} \quad \textit{Ans.}\text{ (a)} \quad \text{[It is probably } N_2O]$$

With k and R known, the speed of sound at 100ºC = 373 K is estimated by

$$a = \sqrt{kRT} = \sqrt{1.31[189\ m^2/(s^2 \cdot K)](373\ K)} \approx \mathbf{304\ m/s} \quad \textit{Ans.}\text{ (b)}$$

P1.38 In Fig. P1.38, if the fluid is glycerin at 20°C and the width between plates is 6 mm, what shear stress (in Pa) is required to move the upper plate at V = 5.5 m/s? What is the flow Reynolds number if "L" is taken to be the distance between plates?

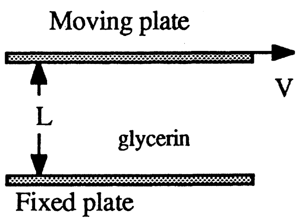


Fig. P1.38

Solution: (a) For glycerin at 20°C, from Table 1.4, $\mu \approx 1.5\ \mathrm{N \cdot s/m^2}$. The shear stress is found from Eq. (1) of Ex. 1.8:

$$\tau = \frac{\mu V}{h} = \frac{(1.5\ \mathrm{Pa \cdot s})(5.5\ \mathrm{m/s})}{(0.006\ \mathrm{m})} \approx \mathbf{1380\ Pa} \quad \textit{Ans.}\ (a)$$

The density of glycerin at 20°C is 1264 $\mathrm{kg/m^3}$. Then the Reynolds number is defined by Eq. (1.24), with L = h, and is found to be decidedly laminar, Re < 1500:

$$Re_L = \frac{\rho VL}{\mu} = \frac{(1264\ \mathrm{kg/m^3})(5.5\ \mathrm{m/s})(0.006\ \mathrm{m})}{1.5\ \mathrm{kg/m \cdot s}} \approx \mathbf{28} \quad \textit{Ans.}\ (b)$$

P1.39 Knowing $\mu \approx 1.80\mathrm{E}{-5}\ \mathrm{Pa \cdot s}$ for air at 20°C from Table 1-4, estimate its viscosity at 500°C by (a) the power-law, (b) the Sutherland law, and (c) the Law of Corresponding States, Fig. 1.5. Compare with the accepted value $\mu(500°\mathrm{C}) \approx 3.58\mathrm{E}{-5}\ \mathrm{Pa \cdot s}$.

Solution: First change T from 500°C to 773 K. (a) For the power-law for air, n ≈ 0.7, and from Eq. (1.30a),

$$\mu = \mu_o (T/T_o)^n \approx (1.80\mathrm{E}{-5})\left(\frac{773}{293}\right)^{0.7} \approx \mathbf{3.55E{-5}\ \frac{kg}{m \cdot s}} \quad \textit{Ans.}\ (a)$$

This is less than 1% low. (b) For the Sutherland law, for air, S ≈ 110 K, and from Eq. (1.30b),

$$\mu = \mu_o \left[\frac{(T/T_o)^{1.5}(T_o + S)}{(T+S)}\right] \approx (1.80\mathrm{E}{-5})\left[\frac{(773/293)^{1.5}(293+110)}{(773+110)}\right]$$

$$= \mathbf{3.52E{-5}\ \frac{kg}{m \cdot s}} \quad \textit{Ans.}\ (b)$$

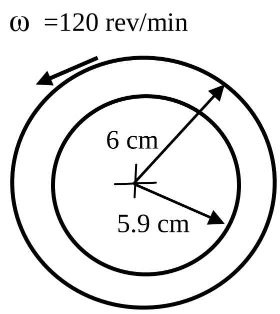
This is only 1.7% low. (c) Finally use Fig. 1.5. Critical values for air from Ref. 3 are:

$$\text{Air:}\quad \mu_c \approx 1.93\text{E}-5\ \text{Pa·s}\quad T_c \approx 132\ \text{K}\quad (\text{“mixture” estimates})$$

At 773 K, the temperature ratio is T/T_C = 773/132 ≈ 5.9. From Fig. 1.5, read μ/μ_C ≈ 1.8. Then our critical-point-correlation estimate of air viscosity is only 3% low:

$$\mu \approx 1.8\mu_c = (1.8)(1.93\text{E–}5) \approx \mathbf{3.5E–5\ \frac{kg}{m{\cdot}s}}\quad \textit{Ans.}\ \text{(c)}$$

P1.40 Glycerin at 20ºC fills the space between a hollow sleeve of diameter12 cm and a fixed coaxial solid rod of diameter 11.8 cm. The outer sleeve is rotated at 120 rev/min. Assuming no temperature change, estimate the torque required, in N·m per meter of rod length, to hold the inner rod fixed.



Solution: From Table A.3, the viscosity of glycerin is 1.49 kg/(m·s). The clearance C is the difference in *radii*, 6 cm – 5.9 cm = 0.1 cm = 1 mm. The velocity of the sleeve surface is $V = \omega r_o$ = [2π(120)/60](0.06) = 0.754 m/s. The shear stressin the glycerin is approximately $\tau = \mu V/C$ = (1.49)(0.754)/(0.001) = 1123 Pa. The shear forces are all perpendicular to the radius and thus have a total torque

$$Torque = \tau(2\pi r_i L)r_i = (1123 N/m^2)[2\pi(0.059m)(1m)](0.059m) = \mathbf{25\,N\cdot m}\ \textit{per meter}\quad \textit{Ans.}$$

P1.41 An aluminum cylinder weighing 30 N, 6 cm in diameter and 40 cm long, is falling concentrically through a long vertical sleeve of diameter 6.04 cm. The clearance is filled with SAE 50 oil at 20°C. Estimate the *terminal* (zero acceleration) fall velocity. Neglect air drag and assume a linear velocity distribution in the oil. [HINT: You are given diameters, not radii.]

Solution: From Table A.3 for SAE 50 oil, μ = 0.86 kg/m-s. The clearance is the difference in *radii*: 3.02 – 3.0 cm = 0.02 cm = 0.0002 m. At terminal velocity, the cylinder weight must balance the viscous drag on the cylinder surface:

$$W = \tau_{wall} A_{wall} = (\mu \frac{V}{C})(\pi DL), \quad \text{where } C = \text{clearance} = r_{sleeve} - r_{cylinder}$$

$$\text{or:} \quad 30\,N = [0.86 \frac{kg}{m-s})(\frac{V}{0.0002\,m})]\,\pi(0.06\,m)(0.40\,m)$$

$$\text{Solve for } V = \mathbf{0.0925}\,m/s \qquad Ans.$$

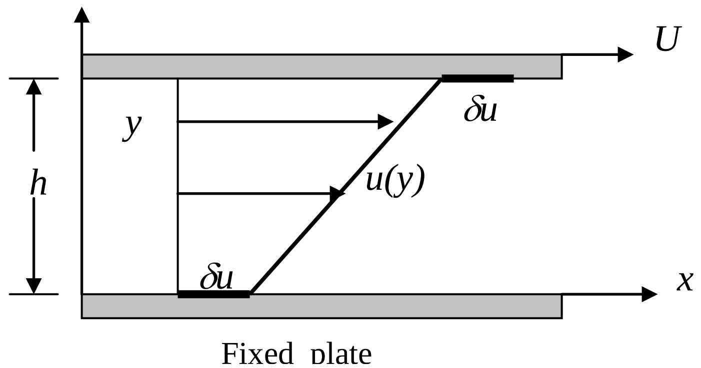
P1.42 Helium at 20ºC has a viscosity of 1.97E-5 kg/(m·s). Use the data of Table A.4 to estimate the temperature, in ºC, at which helium's viscosity will double.

Solution: Table A.4 has no information about the Sutherland constant, but it does recommend a power-law exponent n = 0.68. Thus, for doubling, we simply write the formula, Eq. (1.27), which requires absolute temperatures:

$$\frac{\mu}{\mu_o} = 2.0 = (\frac{T_{doubling}}{293\,K})^{0.68}; \quad solve \quad T_{doubling} \approx 812\,K \approx 539^{o}C \qquad Ans.$$

P1.43 For the flow between two parallel plates of Fig. 1.8, reanalyze for the case of *slip flow* at both walls. Use the simple slip condition, $u_{wall} = \ell\,(du/dy)_{wall}$, where ℓ is the mean free path of the fluid. (*a*) Sketch the expected velocity profile and (*b*) find an expression for the shear stress at each wall.

Solution: As in Fig. 1.8, the shear stress remains constant between the two plates. The analysis is correct up to the relation $u = a + b\,y$. There would be equal slip velocities, δu, at both walls, as shown in the following sketch



Ans. (*a*)

Because of slip at the walls, the boundary conditions are different for $u = a + b\,y$:

$$\text{At } y = 0: \quad u = \delta u = a = \ell\left(\frac{du}{dy}\right)_{y=0} = \ell\, b$$

$$At\ y = h: \quad u = U - \delta u = a + b\,h = U - \ell\, b = \ell\, b + b\,h\ ,\ or:\ b = \frac{U}{h+2\ell}$$

One way to write the final solution is

$$u = \delta u + \left(\frac{U}{h+2\ell}\right) y \quad , \quad where \quad \delta u = \left(\frac{\ell}{h+2\ell}\right) U \quad ,$$

$$and \quad \tau_{wall} = \mu\left(\frac{du}{dy}\right)_{wall} = \frac{\mu U}{h+2\ell} \qquad Ans.(b)$$

P1.44 A popular viscometer is simply a long capillary tube. A commercial device is shown in Prob. C1.10. One measures the volume flow rate Q and the pressure drop Δp and, of course, the radius and length of the tube. The theoretical formula, which will be discussed in Chap. 6, is $\Delta p \approx 8\mu QL/(\pi R^4)$. For a capillary of diameter 4 mm and length 10 inches, the test fluid flows at 0.9 m^3/h when the pressure drop is 58 lbf/in^2. Find the predicted viscosity in kg/m·s.

Solution: Convert everything to SI units: Q = 0.9/3600 = 0.00025 m^3/s, Δp = 58x6894.8 = 400,000 Pa, L = 10/12x0.3048 = 0.254 m, R = D/2 = 0.002 m. Then apply the theoretical formula:

$$\Delta p = 400,000 = 8\mu QL/(\pi R^4) = 8\mu(0.00025)(0.254)/[\pi(0.002)^4]$$

Solve for μ = 0.0396 kg/m·s ≈ **0.040 kg/m·s** *Ans.*

If the density is, say, 900 kg/m^3, the Reynolds number Re_D is about 1800, or below transition.

P1.45 A block of weight W slides down an inclined plane on a thin film of oil, as in Fig. P1.45 at right. The film contact area is A and its thickness h. Assuming a linear velocity distribution in the film, derive an analytic expression for the terminal velocity V of the block.

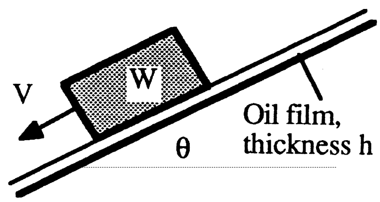


Fig. P1.45

Solution: Let "x" be down the incline, in the direction of V. By "terminal" velocity we mean that there is no acceleration. Assume a linear viscous velocity distribution in the film below the block. Then a force balance in the x direction gives:

$$\sum F_x = W\sin\theta - \tau A = W\sin\theta - \left(\mu\frac{V}{h}\right)A = ma_x = 0,$$

$$\text{or:}\quad V_{terminal} = \frac{\mathbf{hW\sin\theta}}{\boldsymbol{\mu}\mathbf{A}} \quad \textit{Ans.}$$

P1.46 A simple and popular model for two non-newtonian fluids in Fig. 1.9*a* is the *power-law*:

$$\tau \quad \approx \quad C\,(\frac{du}{dy})^n$$

where *C* and *n* are constants fit to the fluid [15]. From Fig. 1.9*a*, deduce the values of the exponent *n* for which the fluid is (*a*) newtonian; (*b*) dilatant; and (*c*) pseudoplastic. (*d*) Consider the specific model constant $C = 0.4\ \text{N-s}^n/\text{m}^2$, with the fluid being sheared between two parallel plates as in Fig. 1.8. If the shear stress in the fluid is 1200 Pa, find the velocity *V* of the upper plate for the cases (*d*) $n = 1.0$; (*e*) $n = 1.2$; and (*f*) $n = 0.8$.

Solution: By comparing the behavior of the model law with Fig. 1.9*a*, we see that
(*a*) Newtonian: $n = 1$; (*b*) Dilatant: $n > 1$; (*c*) Pseudoplastic: $n < 1$ *Ans*.(*a,b,c*)
From the discussion of Fig. 1.8, it is clear that the shear stress is *constant* in a fluid sheared between two plates. The velocity profile remains a straight line (if the flow is laminar), and the strain rate $du/dy = V/h$. Thus, for this flow, the model becomes $\tau = C(V/h)^n$. For the three given numerical cases, we calculate:

$$(d)\ n = 1:\ \tau = 1200\frac{N}{m^2} = C(V/h)^n = (0.4\frac{N-s^1}{m^2})(\frac{V}{0.001m})^1\ ,\quad \text{solve}\quad V = \mathbf{3.0}\frac{\mathbf{m}}{\mathbf{s}}\quad Ans.(d)$$

$$(e)\ n = 1.2:\ \tau = 1200\frac{N}{m^2} = C(V/h)^n = (0.4\frac{N-s^{1.2}}{m^2})(\frac{V}{0.001m})^{1.2},\quad \text{solve}\quad V = \mathbf{0.79}\frac{\mathbf{m}}{\mathbf{s}}\quad Ans.(e)$$

$$(f)\ n = 0.8:\ \tau = 1200\frac{N}{m^2} = C(V/h)^n = (0.4\frac{N-s^{0.8}}{m^2})(\frac{V}{0.001m})^{0.8}\ ,\quad \text{solve}\quad V = \mathbf{22}\frac{\mathbf{m}}{\mathbf{s}}\quad Ans.(f)$$

A small change in the exponent *n* can sharply change the numerical values.

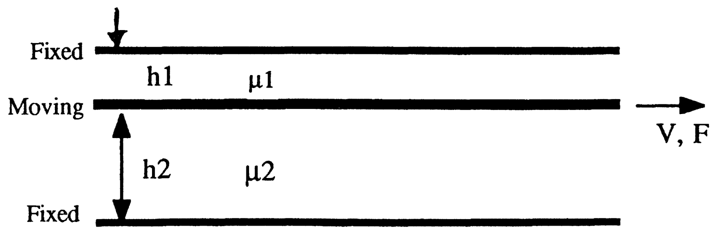
P1.47 Data for the apparent viscosity of average human blood, at normal body temperature of 37°C, varies with shear strain rate, as shown in the following table.

Shear strain rate, s^{-1}	1	10	100	1000
Apparent viscosity, kg/(m•s)	0.011	0.009	0.006	0.004

(*a*) Is blood a nonnewtonian fluid? (*b*) If so, what type of fluid is it? (*c*) How do these viscosities compare with plain water at 37°C?

Solution: (*a*) By definition, since viscosity varies with strain rate, blood is a *nonnewtonian flui*d.
(*b*) Since the apparent viscosity decreases with strain rate, it must be a *pseudoplastic* fluid, as in Fig. 1.9(*a*). The decrease is too slight to call this a "plastic" fluid. (*c*) These viscosity values are from six to fifteen times the viscosity of pure water at 37°C, which is about 0.00070 kg/m-s. The viscosity of the liquid part of blood, called *plasma*, is about 1.8 times that of water. Then there is a sharp increase of blood viscosity due to *hematocrit,* which is the percentage, by volume, of red cells and platelets in the blood. For normal human beings, the hematocrit varies from 40% to 60%, which makes this blood about six times the viscosity of plasma.

P1.48 A thin moving plate is separated from two fixed plates by two fluids of unequal viscosity and unequal spacing, as shown below. The contact area is A. Determine (a) the force required, and (b) is there a necessary relation between the two viscosity values?



Solution: (a) Assuming a linear velocity distribution on each side of the plate, we obtain

$$F = \tau_1 A + \tau_2 A = \left(\frac{\mu_1 V}{h_1} + \frac{\mu_2 V}{h_2}\right) A \quad \textit{Ans. (a)}$$

The formula is of course valid only for laminar (nonturbulent) steady viscous flow.
(b) Since the center plate separates the two fluids, they may have separate, unrelated shear stresses, and there is <u>*no necessary relation*</u> between the two viscosities.

P1.49 An amazing number of commercial and laboratory devices have been developed to measure fluid viscosity, as described in Ref. 27. Consider a concentric shaft, fixed axially and rotated inside the sleeve. Let the inner and outer cylinders have radii r_i and r_o, respectively, with total sleeve length L. Let the rotational rate be Ω (rad/s) and the applied torque be M. Using these parameters, derive a theoretical relation for the viscosity μ of the fluid between the cylinders.

Solution: Assuming a linear velocity distribution in the annular clearance, the shear stress is

$$\tau = \mu\frac{\Delta V}{\Delta r} \approx \mu\frac{\Omega r_i}{r_o - r_i}$$

This stress causes a force $dF = \tau\, dA = \tau(r_i\, d\theta)L$ on each element of surface area of the inner shaft. The moment of this force about the shaft axis is $dM = r_i\, dF$. Put all this together:

$$M = \int r_i\, dF = \int_0^{2\pi} r_i \mu \frac{\Omega r_i}{r_o - r_i} r_i\, L\, d\theta = \frac{2\pi\mu\Omega r_i^3 L}{r_o - r_i}$$

Solve for the viscosity: $\boldsymbol{\mu \approx M(r_o - r_i)\big/\left\{2\pi\Omega r_i^3 L\right\}}$ *Ans.*

P1.50 A simple viscometer measures the time t for a solid sphere to fall a distance L through a test fluid of density ρ. The fluid viscosity μ is then given by

$$\mu \approx \frac{W_{net}\, t}{3\pi DL} \quad if \quad t \ge \frac{2\rho DL}{\mu}$$

where D is the sphere diameter and W_{net} is the sphere net weight in the fluid.
(a) Show that both of these formulas are dimensionally homogeneous.
(b) Suppose that a 2.5 mm diameter aluminum sphere (density 2700 kg/m^3) falls in an oil of density 875 kg/m^3. If the time to fall 50 cm is 32 s, estimate the oil viscosity and verify that the inequality is valid.

Solution: (a) Test the dimensions of each term in the two equations:

$$\{\mu\} = \left\{\frac{M}{LT}\right\} \quad \text{and} \quad \left\{\frac{W_{net}\, t}{(3\pi)DL}\right\} = \left\{\frac{(ML/T^2)(T)}{(1)(L)(L)}\right\} = \left\{\frac{M}{LT}\right\} \quad \textbf{Yes}, \text{ dimensions OK.}$$

$$\{t\} = \{T\} \quad \text{and} \quad \left\{\frac{2\rho DL}{\mu}\right\} = \left\{\frac{(1)(M/L^3)(L)(L)}{M/LT}\right\} = \{T\} \quad \textbf{Yes}, \text{ dimensions OK.} \quad \textit{Ans. (a)}$$

(b) Evaluate the two equations for the data. We need the net weight of the sphere in the fluid:

$$W_{net} = (\rho_{sphere} - \rho_{fluid})g(Vol)_{fluid} = (2700 - 875\ \text{kg/m}^3)(9.81\ \text{m/s}^2)(\pi/6)(0.0025\ \text{m})^3$$
$$= 0.000146\ \text{N}$$

$$\text{Then}\ \ \mu = \frac{W_{net}t}{3\pi DL} = \frac{(0.000146\ N)(32\ s)}{3\pi(0.0025\ m)(0.5\ m)} = \mathbf{0.40}\ \frac{kg}{m\cdot s} \quad \textit{Ans.}\text{ (b)}$$

$$\text{Check } t = 32\ s \text{ compared to } \frac{2\rho DL}{\mu} = \frac{2(875\ kg/m^3)(0.0025\ m)(0.5\ m)}{0.40\ kg/m\cdot s}$$
$$= 5.5\ s \ \text{ OK, } t \text{ is greater}$$

P1.51 An approximation for the boundary-layer shape in Figs. 1.6*b* and P1.51 is the formula

$$u(y) \approx U\sin\left(\frac{\pi y}{2\delta}\right), \quad 0 \le y \le \delta$$

where U is the stream velocity far from the wall and δ is the boundary layer thickness, as in Fig. P.151. If the fluid is helium at 20°C and 1 atm, and if U = 10.8 m/s and δ = 3 mm, use the formula to (*a*) estimate the wall shear stress τ_w in Pa; and (*b*) find the position in the boundary layer where τ is one-half of τ_w.

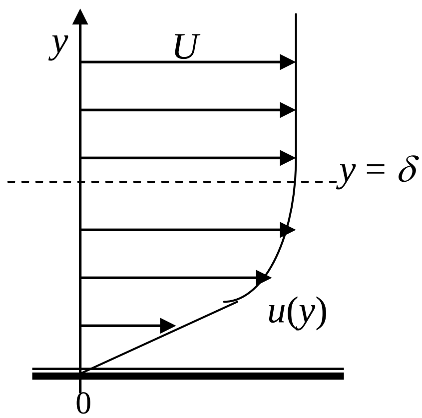


Fig. P1.51

Solution: From Table A.4, for helium, take R = 2077 m²/(s²-K) and μ = 1.97E-5 kg/m-s. (*a*) Then the wall shear stress is calculated as

$$\tau_w = \mu\frac{\partial u}{\partial y}\bigg|_{y=0} = \mu\left(U\frac{\pi}{2\delta}\cos\frac{\pi y}{2\delta}\right)_{y=0} = \frac{\pi\,\mu U}{2\,\delta}$$

$$\text{Numerical values: } \tau_w = \frac{\pi(1.97E-5\,kg/m-s)(10.8\,m/s)}{2(0.003\,m)} = \mathbf{0.11\ Pa} \qquad \textit{Ans.}(a)$$

A very small shear stress, but it has a profound effect on the flow pattern.

(*b*) The variation of shear stress across the boundary layer is a cosine wave, $\tau = \mu(du/dy)$:

$$\tau(y) = \frac{\pi\mu U}{2\delta}\cos\left(\frac{\pi y}{2\delta}\right) = \tau_w \cos\left(\frac{\pi y}{2\delta}\right) = \frac{\tau_w}{2} \text{ when } \frac{\pi y}{2\delta} = \frac{\pi}{3}, \text{ or: } \mathbf{y} = \frac{\mathbf{2\delta}}{\mathbf{3}} \quad \textit{Ans.}(b)$$

P1.52 The belt in Fig. P1.52 moves at steady velocity V and skims the top of a tank of oil of viscosity μ. Assuming a linear velocity profile, develop a simple formula for the belt-drive power P required as a function of (h, L, V, B, μ). Neglect air drag. What power P in watts is required if the belt moves at 2.5 m/s over SAE 30W oil at 20°C, with L = 2 m, b = 60 cm, and h = 3 cm?

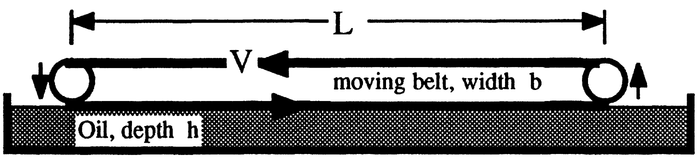


Fig. P1.52

Solution: The power is the viscous resisting force times the belt velocity:

$$P = \tau_{oil}A_{belt}V_{belt} \approx \left(\mu\frac{V}{h}\right)(bL)V = \boldsymbol{\mu V^2 b\frac{L}{h}} \quad \textit{Ans.}$$

(b) For SAE 30W oil, $\mu \approx 0.29$ kg/m·s. Then, for the given belt parameters,

$$P = \mu V^2 bL/h = \left(0.29\ \frac{kg}{m\cdot s}\right)\left(2.5\ \frac{m}{s}\right)^2 (0.6\ m)\frac{2.0\ m}{0.03\ m} \approx 73\ \frac{kg\cdot m^2}{s^3} = \mathbf{73\ W} \quad \textit{Ans.}\ (b)$$

***P1.53** A solid cone of base r_o and initial angular velocity ω_o is rotating inside a conical seat. Neglect air drag and derive a formula for the cone's angular velocity $\omega(t)$ if there is no applied torque.

Solution: At any radial position $r < r_o$ on the cone surface and instantaneous rate ω,

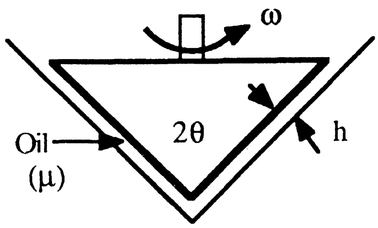


Fig. P1.53

$$d(\text{Torque}) = r\tau\, dA_w = r\left(\mu\frac{r\omega}{h}\right)\left(2\pi r\frac{dr}{\sin\theta}\right),$$

$$\text{or:} \quad \text{Torque } M = \int_0^{r_o} \frac{\mu\omega}{h\sin\theta} 2\pi r^3\, dr = \frac{\pi\mu\omega r_o^4}{2h\sin\theta}$$

We may compute the cone's slowing down from the angular momentum relation:

$$M = -I_o\frac{d\omega}{dt}, \quad \text{where } I_o(\text{cone}) = \frac{3}{10}mr_o^2, \quad m = \text{cone mass}$$

Separating the variables, we may integrate:

$$\int_{\omega_o}^{w} \frac{d\omega}{\omega} = -\frac{\pi\mu r_o^4}{2hI_o\sin\theta}\int_0^t dt, \quad \text{or:} \quad \boldsymbol{\omega = \omega_o \exp\left[-\frac{5\pi\mu r_o^2 t}{3mh\sin\theta}\right]} \quad \textit{Ans.}$$

***P1.54** A disk of radius R rotates at angular velocity Ω inside an oil container of viscosity μ, as in Fig. P1.54. Assuming a linear velocity profile and neglecting shear on the outer disk edges, derive an expression for the viscous torque on the disk.

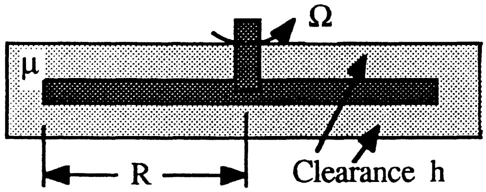


Fig. P1.54

Solution: At any r ≤ R, the viscous shear $\tau \approx \mu\Omega r/h$ on both sides of the disk. Thus,

$$d(\text{torque}) = dM = 2r\tau\, dA_w = 2r\frac{\mu\Omega r}{h}2\pi r\, dr,$$

$$\text{or:}\quad M = 4\pi\frac{\mu\Omega}{h}\int_0^R r^3\, dr = \frac{\pi\mu\Omega R^4}{h} \quad \textit{Ans.}$$

P1.55 A block of weight W is being pulled over a table by another weight W_o, as shown in Fig. P1.55. Find an algebraic formula for the steady velocity U of the block if it slides on an oil film of thickness h and viscosity μ. The block bottom area A is in contact with the oil. Neglect the cord weight and the pulley friction.

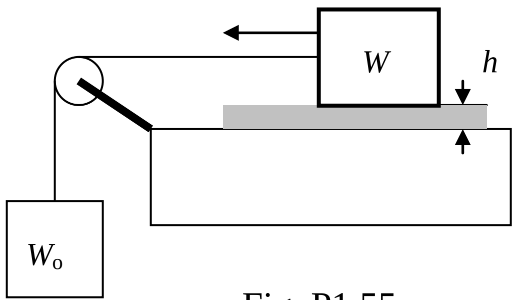


Fig. P1.55

Solution: This problem is a lot easier to *solve* than to set up and sketch. For steady motion, there is no acceleration, and the falling weight balances the viscous resistance of the oil film:

$$\sum F_{x,block} = 0 = \tau A - W_o = (\mu\frac{U}{h})A - W_o$$

$$\text{Solve for}\quad U = \frac{W_o h}{\mu A} \qquad \textit{Ans.}$$

The block weight W has no effect on steady horizontal motion except to smush the oil film.

***P1.56** For the cone-plate viscometer in Fig. P1.56, the angle is very small, and the gap is filled with test liquid μ. Assuming a linear velocity profile, derive a formula for the viscosity μ in terms of the torque M and cone parameters.

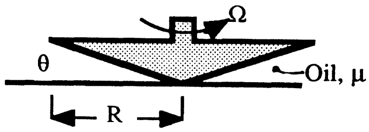


Fig. P1.56

Solution: For any radius r ≤ R, the liquid gap is h = r tanθ. Then

$$d(\text{Torque}) = dM = \tau\, dA_w r = \left(\mu \frac{\Omega r}{r\tan\theta}\right)\left(2\pi r \frac{dr}{\cos\theta}\right) r, \quad \text{or}$$

$$M = \frac{2\pi\Omega\mu}{\sin\theta}\int_0^R r^2\,dr = \frac{2\pi\Omega\mu R^3}{3\sin\theta}, \quad \text{or:} \quad \mu = \frac{3M\sin\theta}{2\pi\Omega R^3} \quad \textit{Ans.}$$

P1.57 Extend the steady flow between a fixed lower plate and a moving upper plate, from Fig. 1.8, to the case of two immiscible liquids between the plates, as in Fig. P1.57.

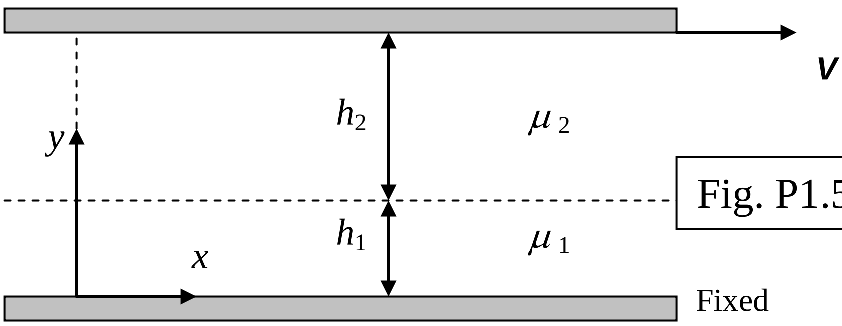
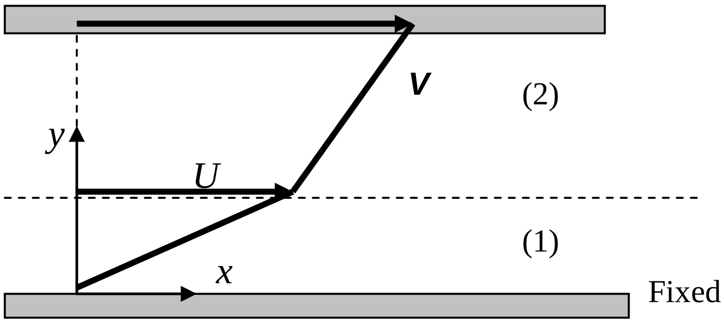


Fig. P1.57

(*a*) Sketch the expected no-slip velocity distribution *u*(*y*) between the plates. (*b*) Find an analytic expression for the velocity *U* at the interface between the two liquid layers. (*c*) What is the result if the viscosities and layer thicknesses are equal?

Solution: We begin with the hint, from Fig. 1.8, that the shear stress is constant between the two plates. The velocity profile would be a straight line in each layer, with different slopes:



Ans. (*a*)

Here we have drawn the case where $\mu_2 > \mu_1$, hence the upper profile slope is less. (*b*) Set the two shear stresses equal, assuming no-slip at each wall:

$$\tau_1 = \mu_1 \frac{(U-0)}{h_1} = \tau_2 = \mu_2 \frac{(V-U)}{h_2}$$

$$\text{Solve for} \quad U = V\left[\frac{1}{1+\dfrac{\mu_1 h_2}{\mu_2 h_1}}\right] \qquad \textit{Ans.}(b)$$

(*c*) For equal viscosities and layer thicknesses, we get the simple result $U = V/2$. *Ans*.

P1.58 The laminar-pipe-flow example of Prob. 1.14 leads to a *capillary viscometer* [29], using the formula $\mu = \pi r_o^4 \Delta p/(8LQ)$. Given r_o = 2 mm and L = 25 cm. The data are

Q, m^3/hr:	0.36	0.72	1.08	1.44	1.80
Δp, kPa:	159	318	477	1274	1851

Estimate the fluid viscosity. What is wrong with the last two data points?

Solution: Apply our formula, with consistent units, to the first data point:

$$\Delta p = 159\,\text{kPa:}\quad \mu \approx \frac{\pi r_o^4 \Delta p}{8LQ} = \frac{\pi(0.002\text{ m})^4(159000\text{ N/m}^2)}{8(0.25\text{ m})(0.36/3600\text{ m}^3/\text{s})} \approx 0.040\ \frac{\text{N}\cdot\text{s}}{\text{m}^2}$$

Do the same thing for all five data points:

Δp, kPa:	159	318	477	1274	1851	
μ, N·s/m^2:	**0.040**	**0.040**	**0.040**	*0.080*(?)	*0.093*(?)	*Ans.*

The last two estimates, though measured properly, are *incorrect*. The Reynolds number of the capillary has risen above 2000 and the flow is *turbulent*, which requires a different formula.

P1.59 A solid cylinder of diameter D, length L, density ρ_s falls due to gravity inside a tube of diameter D_o. The clearance, $(D_o - D) << D$, is filled with a film of viscous fluid (ρ,μ). Derive a formula for terminal fall velocity and apply to SAE 30 oil at 20°C for a steel cylinder with D = 2 cm, D_o = 2.04 cm, and L = 15 cm. Neglect the effect of any air in the tube.

Solution: The geometry is similar to Prob. 1.47, only vertical instead of horizontal. At terminal velocity, the cylinder weight should equal the viscous drag:

$$a_z = 0:\quad \Sigma F_z = -W + \text{Drag} = -\rho_s g \frac{\pi}{4} D^2 L + \left[\mu \frac{V}{(D_o - D)/2}\right]\pi DL,$$

$$\text{or:}\quad V = \boldsymbol{\frac{\rho_s g D(D_o - D)}{8\mu}}\quad \textit{Ans.}$$

For the particular numerical case given, $\rho_{steel} \approx 7850$ kg/m^3. For SAE 30 oil at 20°C, $\mu \approx 0.29$ kg/m·s from Table 1.4. Then the formula predicts

$$V_{terminal} = \frac{\rho_s g D(D_o - D)}{8\mu} = \frac{(7850\text{ kg/m}^3)(9.81\text{ m/s}^2)(0.02\text{ m})(0.0204 - 0.02\text{ m})}{8(0.29\text{ kg/m}\cdot\text{s})}$$

$$\approx \mathbf{0.265\ m/s}\quad \textit{Ans.}$$

P1.60 Pipelines are cleaned by pushing through them a close-fitting cylinder called a *pig*. The name comes from the squealing noise it makes sliding along. Ref. 50 describes a new non-toxic pig, driven by compressed air, for cleaning cosmetic and beverage pipes. Suppose the pig diameter is 5-15/16 in and its length 26 in. It cleans a 6-in-diameter pipe at a speed of 1.2 m/s. If the clearance is filled with glycerin at 20°C, what pressure difference, in pascals, is needed to drive the pig? Assume a linear velocity profile in the oil and neglect air drag.

Solution: Since the problem calls for *pascals*, convert everything to SI units:
Find the shear stress in the oil, multiply that by the cylinder wall area to get the required force, and divide the force by the area of the cylinder face to find the required pressure difference.

Comment: The Reynolds number of the clearance flow, Re = $\rho VC/\mu$, is approximately 0.8.

$$D_{cyl} = (5\tfrac{15}{16}\,in)(0.0254\frac{m}{in}) = 0.1508\,m \quad ; \quad L = (26\,in)(0.0254\frac{m}{in}) = 0.6604\,m$$

$$\text{Clearance} = C = (D_{pipe} - D_{cyl})/2 = (6 - 5\tfrac{15}{16}\,in)(0.0254\frac{m}{in})/2 = 0.000794\,m$$

$$\text{Table A.3, glycerin:} \quad \mu = 1.49\ kg/m-s \ , \quad \rho = 1260\ kg/m^3 \ \ (\rho \text{ not needed})$$

$$\text{Shear stress } \tau = \mu V/C = (1.49\ kg/m-s)(1.2\,m/s)/0.000794\,m = 2252\,N/m^2$$

$$\text{Shear force } F = \tau A_{wall} = \tau(\pi D_{cyl} L) =$$

$$(2252\,N/m^2)[\pi(0.1508\,m)(0.6604\,m)] = 705\,N$$

$$\text{Finally,} \quad \Delta p = \frac{F}{A_{cyl\ face}} = \frac{705\,N}{(\pi/4)(0.1508\,m)^2} = \mathbf{39500\ Pa} \qquad \textit{Ans.}$$

P1.61 An air-hockey puck has m = 50 g and D = 9 cm. When placed on a 20°C air table, the blower forms a 0.12-mm-thick air film under the puck. The puck is struck with an initial velocity of 10 m/s. How long will it take the puck to (a) slow down to 1 m/s; (b) stop completely? Also (c) how far will the puck have travelled for case (a)?

Solution: For air at 20°C take $\mu \approx$ 1.8E−5 kg/m·s. Let A be the bottom area of the puck, $A = \pi D^2/4$. Let x be in the direction of travel. Then the only force acting in the x direction is the air drag resisting the motion, assuming a linear velocity distribution in the air:

$$\sum F_x = -\tau A = -\mu\frac{V}{h}A = m\frac{dV}{dt}, \quad \text{where h = air film thickness}$$

Separate the variables and integrate to find the velocity of the decelerating puck:

$$\int_{V_o}^{V}\frac{dV}{V} = -K\int_0^t dt, \quad \text{or} \quad V = V_o e^{-Kt}, \quad \text{where } K = \frac{\mu A}{mh}$$

Integrate again to find the displacement of the puck:

$$x = \int_0^t V\,dt = \frac{V_o}{K}[1 - e^{-Kt}]$$

Apply to the particular case given: air, $\mu \approx$ 1.8E–5 kg/m·s, m = 50 g, D = 9 cm, h = 0.12 mm, V_o = 10 m/s. First evaluate the time-constant K:

$$K = \frac{\mu A}{mh} = \frac{(1.8E\text{–}5\ \text{kg/m}\cdot\text{s})[(\pi/4)(0.09\ \text{m})^2]}{(0.050\ \text{kg})(0.00012\ \text{m})} \approx 0.0191\ \text{s}^{-1}$$

(a) When the puck slows down to 1 m/s, we obtain the time:

$$V = 1\ \text{m/s} = V_o e^{-Kt} = (10\ \text{m/s})\,e^{-(0.0191\ \text{s}^{-1})t}, \quad \text{or} \quad t \approx \mathbf{121\ s} \quad \textit{Ans.}\ \text{(a)}$$

(b) The puck will stop completely only when $\mathbf{e^{-Kt} = 0}$**, or:** $\mathbf{t = \infty}$ *Ans*. (b)
(c) For part (a), the puck will have travelled, in 121 seconds,

$$x = \frac{V_o}{K}(1 - e^{-Kt}) = \frac{10\ \text{m/s}}{0.0191\ \text{s}^{-1}}[1 - e^{-(0.0191)(121)}] \approx \mathbf{472\ m} \quad \textit{Ans.}\ \text{(c)}$$

This may perhaps be a little unrealistic. But the air-hockey puck *does* decelerate slowly!

P1.62 The hydrogen bubbles in Fig. 1.13 have D ≈ 0.01 mm. Assume an "air-water" interface at 30°C. What is the excess pressure within the bubble?

Solution: At 30°C the surface tension from Table A-1 is 0.0712 N/m. For a droplet or bubble with one spherical surface, from Eq. (1.32),

$$\Delta p = \frac{2Y}{R} = \frac{2(0.0712\text{ N/m})}{(5E\text{–}6\text{ m})} \approx \mathbf{28500\ Pa} \quad \textit{Ans.}$$

P1.63 Derive Eq. (1.37) by making a force balance on the fluid interface in Fig. 1.9c.

Solution: The surface tension forces YdL1 and YdL2 have a slight vertical component. Thus summation of forces in the vertical gives the result

$$\sum F_z = 0 = 2YdL_2 \sin(d\theta_1/2) + 2YdL_1 \sin(d\theta_2/2) - \Delta p\, dA$$

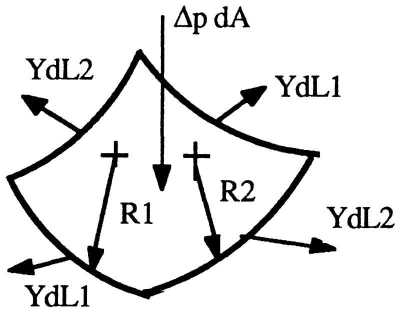


Fig. 1.9c

But dA = dL1dL2 and sin(dθ/2) ≈ dθ/2, so we may solve for the pressure difference:

$$\Delta p = Y\frac{dL_2 d\theta_1 + dL_1 d\theta_2}{dL_1 dL_2} = Y\left(\frac{d\theta_1}{dL_1} + \frac{d\theta_2}{dL_2}\right) = \mathbf{Y}\left(\frac{\mathbf{1}}{\mathbf{R_1}} + \frac{\mathbf{1}}{\mathbf{R_2}}\right) \quad \textit{Ans.}$$

P1.64 Pressure in a water container can be measured by an open vertical tube called a piezometer – see Fig. P2.11 for a typical sketch. If the expected water rise is about 20 cm, what tube diameter is needed to ensure that the error due to capillarity will be less than 3 per cent?

Solution: For water on glass, take Υ = 0.073 N/m and θ = 0º. We want the capillary rise *h* to be less than 3% of 20 cm, or about 0.006 m. The appropriate formula was developed in Ex. 1.8:

$$h = 0.006m = \frac{2\Upsilon\cos\theta}{\rho g R} = \frac{2(0.073N/m)\cos(0°)}{(998kg/m^3)(9.81m/s^2)R}, \; solve\ for\ \ R = 0.0025m$$

Thus the desired tube diameter *D* = 2*R* should be greater than 0.005 m = **5 mm** *Ans.*

1.65 The system in Fig. P1.65 is used to estimate the pressure p_1 in the tank by measuring the 15-cm height of liquid in the 1-mm-diameter tube. The fluid is at 60°C. Calculate the true fluid height in the tube and the percent error due to capillarity if the fluid is (a) water; and (b) mercury.

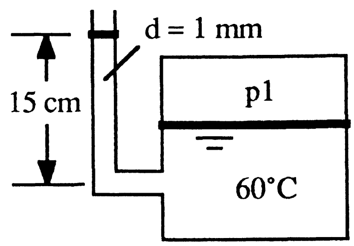


Fig. P1.65

Solution: This is a somewhat more realistic variation of Ex. 1.9. Use values from that example for contact angle θ:

(a) Water at 60°C: $\gamma \approx 9640\ \text{N/m}^3$, $\theta \approx 0°$:

$$h = \frac{4Y\cos\theta}{\gamma D} = \frac{4(0.0662\ \text{N/m})\cos(0°)}{(9640\ \text{N/m}^3)(0.001\ \text{m})} = 0.0275\ \text{m},$$

or: $\Delta h_{true} = 15.0 - 2.75\ \text{cm} \approx$ **12.25 cm (+22% error)** *Ans.* (a)

(b) Mercury at 60°C: $\gamma \approx 132200\ \text{N/m}^3$, $\theta \approx 130°$:

$$h = \frac{4Y\cos\theta}{\gamma D} = \frac{4(0.47\ \text{N/m})\cos 130°}{(132200\ \text{N/m}^3)(0.001\ \text{m})} = -0.0091\ \text{m},$$

or: $\Delta h_{true} = 15.0 + 0.91 \approx \mathbf{15.91\,cm\,(-6\%\,error)}$ *Ans.* (b)

1.66 A thin wire ring, 3 cm in diameter, is lifted from a water surface at 20°C. What is the lift force required? Is this a good method? Suggest a ring material.

Solution: In the literature this ring-pull device is called a DuNouy Tensiometer. The forces are very small and may be measured by a calibrated soft-spring balance. Platinum-iridium is recommended for the ring, being noncorrosive and highly wetting to most liquids. There are two surfaces, inside and outside the ring, so the total force measured is

$$F = 2(Y\pi D) = 2Y\pi D$$

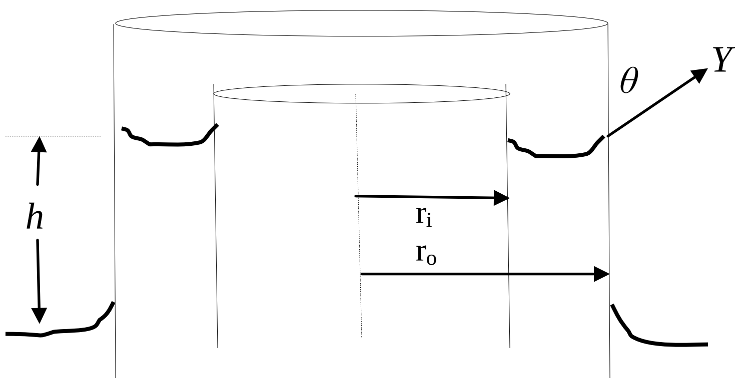
This is crude—commercial devices recommend multiplying this relation by a correction factor $f = O(1)$ which accounts for wire diameter and the distorted surface shape.

For the given data, $Y \approx 0.0728$ N/m (20°C water/air) and the estimated pull force is

$$F = 2\pi(0.0728 \text{ N/m})(0.03 \text{ m}) \approx \mathbf{0.0137\ N} \quad \textit{Ans.}$$

For further details, see, e.g., F. Daniels et al., *Experimental Physical Chemistry*, 7th ed., McGraw-Hill Book Co., New York, 1970.

1.67 A vertical concentric annulus, with outer radius r_o and inner radius r_i, is lowered into fluid of surface tension Y and contact angle $\theta < 90°$. Derive an expression for the capillary rise h in the annular gap, if the gap is very narrow.



Solution: For the figure above, the force balance on the annular fluid is

$$Y\cos\theta(2\pi r_o + 2\pi r_i) = \rho g \pi \left(r_o^2 - r_i^2\right)h$$

Cancel where possible and the result is

$$\boldsymbol{h = 2Y\cos\theta/\{\rho g(r_o - r_i)\}} \quad \textit{Ans.}$$

***P1.68** Analyze the shape $\eta(x)$ of the water-air interface near a wall, as shown. Assume small slope, $R^{-1} \approx d^2\eta/dx^2$. The pressure difference across the interface is $\Delta p \approx \rho g \eta$, with a contact angle θ at x = 0 and a horizontal surface at x = ∞. Find an expression for the maximum height *h*.

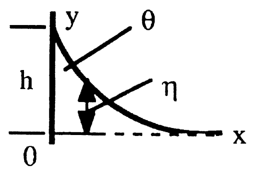


Fig. P1.68

Solution: This is a two-dimensional surface-tension problem, with single curvature. The surface tension rise is balanced by the weight of the film. Therefore the differential equation is

$$\Delta p = \rho g \eta = \frac{Y}{R} \approx Y\frac{d^2\eta}{dx^2} \quad \left(\frac{d\eta}{dx} << 1\right)$$

This is a second-order differential equation with the well-known solution,

$$\eta = C_1 \exp[Kx] + C_2 \exp[-Kx], \quad K = \sqrt{(\rho g/Y)}$$

To keep η from going infinite as x = ∞, it must be that $\mathbf{C_1 = 0}$. The constant C_2 is found from the maximum height at the wall:

$$\eta|_{x=0} = h = C_2 \exp(0), \quad \text{hence } C_2 = h$$

Meanwhile, the contact angle shown above must be such that,

$$\frac{d\eta}{dx}\bigg|_{x=0} = -\cot(\theta) = -hK, \quad \text{thus } h = \frac{\cot\theta}{K}$$

The complete (small-slope) solution to this problem is:

$$\boldsymbol{\eta = h\exp[-(\rho g/Y)^{1/2}x]}, \quad \text{where } h = (Y/\rho g)^{1/2}\cot\theta \quad \textit{Ans.}$$

The formula clearly satisfies the requirement that $\eta = 0$ if x = ∞. It requires "small slope" and therefore the contact angle should be in the range 70° < θ < 110°.

P1.69 A solid cylindrical needle of diameter d, length L, and density ρ_n may "float" on a liquid surface. Neglect buoyancy and assume a contact angle of 0°. Calculate the maxi-mum diameter needle able to float on the surface.

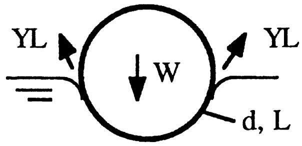


Fig. P1.69

Solution: The needle "dents" the surface downward and the surface tension forces are upward, as shown. If these tensions are nearly vertical, a vertical force balance gives:

$$\sum F_z = 0 = 2YL - \rho g \frac{\pi}{4} d^2 L, \quad \text{or:} \quad \mathbf{d_{max} \approx \sqrt{\frac{8Y}{\pi \rho g}}} \quad \textit{Ans. } \text{(a)}$$

(b) Calculate d_{max} for a steel needle (SG ≈ 7.84) in water at 20°C. The formula becomes:

$$d_{max} = \sqrt{\frac{8Y}{\pi \rho g}} = \sqrt{\frac{8(0.073 \text{ N/m})}{\pi(7.84 \times 998 \text{ kg/m}^3)(9.81 \text{ m/s}^2)}} \approx 0.00156 \text{ m} \approx \mathbf{1.6 \text{ mm}} \quad \textit{Ans. } \text{(b)}$$

P1.70 Derive an expression for the capillary-height change h, as shown, for a fluid of surface tension Y and contact angle θ between two parallel plates W apart. Evaluate h for water at 20°C if W = 0.5 mm.

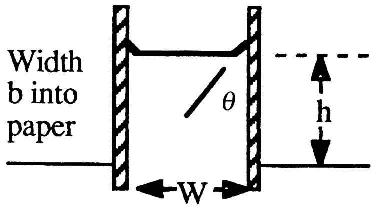


Fig. P1.70

Solution: With b the width of the plates into the paper, the capillary forces on each wall together balance the weight of water held above the reservoir free surface:

$$\rho g W h b = 2(Y b \cos\theta), \quad \text{or:} \quad h \approx \mathbf{\frac{2Y\cos\theta}{\rho g W}} \quad \textit{Ans.}$$

For water at 20°C, Y ≈ 0.0728 N/m, ρg ≈ 9790 N/m^3, and θ ≈ 0°. Thus, for W = 0.5 mm,

$$h = \frac{2(0.0728 \text{ N/m})\cos 0°}{(9790 \text{ N/m}^3)(0.0005 \text{ m})} \approx 0.030 \text{ m} \approx \mathbf{30 \text{ mm}} \quad \textit{Ans.}$$

***P1.71** A soap bubble of diameter D_1 coalesces with another bubble of diameter D_2 to form a single bubble D_3 with the same amount of air. For an isothermal process, express D_3 as a function of D_1, D_2, p_{atm}, and surface tension Y.

Solution: The masses remain the same for an isothermal process of an ideal gas:

$$m_1 + m_2 = \rho_1\upsilon_1 + \rho_2\upsilon_2 = m_3 = \rho_3\upsilon_3,$$

$$\text{or: } \left(\frac{p_a + 4Y/r_1}{RT}\right)\left(\frac{\pi}{6}D_1^3\right) + \left(\frac{p_a + 4Y/r_2}{RT}\right)\left(\frac{\pi}{6}D_2^3\right) = \left(\frac{p_a + 4Y/r_3}{RT}\right)\left(\frac{\pi}{6}D_3^3\right)$$

The temperature cancels out, and we may clean up and rearrange as follows:

$$\mathbf{p_a D_3^3 + 8YD_3^2 = \left(p_a D_2^3 + 8YD_2^2\right) + \left(p_a D_1^3 + 8YD_1^2\right)} \quad \textit{Ans.}$$

This is a cubic polynomial with a known right hand side, to be solved for D_3.

P1.72 Early mountaineers boiled water to estimate their altitude. If they reach the top and find that water boils at 84°C, approximately how high is the mountain?

Solution: From Table A-5 at 84°C, vapor pressure $p_v \approx 55.4$ kPa. We may use this value to interpolate in the standard altitude, Table A-6, to estimate

$$z \approx \mathbf{4800\ m} \quad \textit{Ans.}$$

P1.73 A small submersible moves at velocity V in 20°C water at 2-m depth, where ambient pressure is 131 kPa. Its critical cavitation number is Ca ≈ 0.25. At what velocity will cavitation bubbles form? Will the body cavitate if V = 30 m/s and the water is cold (5°C)?

Solution: From Table A-5 at 20°C read $p_v = 2.337$ kPa. By definition,

$$Ca_{crit} = 0.25 = \frac{2(p_a - p_v)}{\rho V^2} = \frac{2(131000 - 2337)}{(998\ kg/m^3)V^2}, \quad \text{solve } V_{crit} \approx \mathbf{32.1\ m/s} \quad \textit{Ans.}\ (a)$$

If we decrease water temperature to 5°C, the vapor pressure reduces to 863 Pa, and the density changes slightly, to 1000 kg/m^3. For this condition, if V = 30 m/s, we compute:

$$Ca = \frac{2(131000 - 863)}{(1000)(30)^2} \approx 0.289$$

This is *greater* than 0.25, therefore the body **will not cavitate for these conditions**. *Ans.* (b)

P1.74 Oil, with a vapor pressure of 20 kPa, is delivered through a pipeline by equally-spaced pumps, each of which increases the oil pressure by 1.3 MPa. Friction losses in the pipe are 150 Pa per meter of pipe. What is the maximum possible pump spacing to avoid cavitation of the oil?

Solution: The absolute maximum length L occurs when the pump inlet pressure is slightly greater than 20 kPa. The pump increases this by 1.3 MPa and friction drops the pressure over a distance L until it again reaches 20 kPa. In other words, quite simply,

$$1.3 \text{ MPa} = 1{,}300{,}000 \text{ Pa} = (150 \text{ Pa/m})L, \quad \text{or} \quad L_{max} \approx \mathbf{8660\ m} \quad \textit{Ans.}$$

It makes more sense to have the pump inlet at 1 atm, not 20 kPa, dropping L to about 8 km.

P1.75 An airplane flies at 555 mi/h. At what altitude in the standard atmosphere will the airplane's Mach number be exactly 0.8?

Solution: First convert V = 555 mi/h x 0.44704 = 248.1 m/s. Then the speed of sound is

$$a = \frac{V}{\text{Ma}} = \frac{248.1\, m/s}{0.8} = 310\, m/s$$

Reading in Table A.6, we estimate the altitude to be approximately **7500 m**. *Ans*.

P1.76 Derive a formula (a) for the bulk modulus β of an ideal gas, with constant specific heats, and (b) calculate it for steam at 300ºC and 200 kPa. (c) Compare your result to experimental data.

Solution: If an ideal gas is isentropic, $p = C\rho^k$, where $k = c_p/c_v$. Evaluate the bulk modulus:

$$\beta = \rho\left(\frac{\partial p}{\partial \rho}\right)_s = \rho\frac{d}{d\rho}(C\rho^k) = \rho Ck\rho^{k-1} = Ck\rho^k = k\,p \qquad \textit{Ans.}$$

From Table A.4, for H_2O, k = 1.33, hence β_{steam} = (1.33)(200,000) = **266,000 Pa** *Ans.*(*b*)

(*c*) The steam tables do not list a bulk modulus, but we can use the identity $\beta \equiv \rho\, a^2$, where a is the speed of sound. We can enter, for example, SpiraxSarco.com, at p = 200 kPa and T = 300ºC, to obtain ρ = 0.7598 kg/m³ and a = 585.88 m/s. Thus, for this temperature and pressure,

$$\beta_{steam} \approx (0.7598)(585.88)^2 = \mathbf{260{,}000\ Pa} \qquad \textit{Ans.}(c)$$

These estimates are reasonably close.

P1.77 Assume that the n-pentane data of Prob. P1.36 represents isentropic conditions. Estimate the value of the speed of sound at a pressure of 30 MPa. [Hint: The data approximately fit Eq. (1.19) with B = 260 and n = 11.]

Solution: We have to differentiate Eq. (1.19) to find $dp/d\rho$, using the given data p_o = 100 kPa, ρ_o = 586.3 kg/m³, B = 260, and n = 11:

$$dp = d[p_o(B+1)(\rho/\rho_o)^n - B],\ or:\ \frac{dp}{d\rho} = \frac{p_o(B+1)n}{\rho_o}\left(\frac{\rho}{\rho_o}\right)^{n-1} = a^2$$

To finish, we have to find ρ for p = 30 MPa = 30,000 kPa:

$$\frac{p}{p_o} = \frac{30{,}000}{100} = (260+1)\left(\frac{\rho}{586.3}\right)^{11} - 260\ ,\ solve\ \ \rho = 628.4\ kg/m^3$$

$$\text{then}\ \ a^2 = \left[\frac{100{,}000(261)11}{586.3}\right]\left(\frac{628.4}{586.3}\right)^{10} = 980{,}200\ \ \text{, solve}\ \ a = \mathbf{990\ m/s} \qquad \textit{Ans.}$$

P1.78 Sir Isaac Newton measured sound speed by timing the difference between seeing a cannon's puff of smoke and hearing its boom. If the cannon is on a mountain 5.2 miles away, estimate the air temperature in °C if the time difference is (a) 24.2 s; (b) 25.1 s.

Solution: Cannon booms are finite (shock) waves and travel slightly faster than sound waves, but what the heck, assume it's close enough to sound speed:

(a) $a \approx \frac{\Delta x}{\Delta t} = \frac{5.2(5280)(0.3048)}{24.2} = 345.8\ \frac{m}{s} = \sqrt{1.4(287)T},\quad T \approx 298\ K \approx \mathbf{25°C}$ *Ans.* (a)

(b) $a \approx \frac{\Delta x}{\Delta t} = \frac{5.2(5280)(0.3048)}{25.1} = 333.4\ \frac{m}{s} = \sqrt{1.4(287)T},\quad T \approx 277\ K \approx \mathbf{4°C}$ *Ans.* (b)

P1.79 From Table A.3, the density of glycerin at standard conditions is about 1260 kg/m³. At a very high pressure of 8000 lb/in², its density increases to approximately 1275 kg/m³. Use this data to estimate the speed of sound of glycerin, in ft/s.

Solution: For a liquid, we simplify Eq. (1.38) to a pressure-density ratio, without knowing if the process is isentropic or not. This should give satisfactory accuracy:

$$a^2\big|_{glycerin} \approx \frac{\Delta p}{\Delta \rho} = \frac{(8000-15\,lb/in^2)(6895\,Pa/psi)}{(1275-1260)\,kg/m^3} = 3.67E6\ \frac{m^2}{s^2}$$

$$Hence\ \ a \approx \sqrt{3.67E6} \approx 1920\,m/s \approx \mathbf{6300}\ ft/s \qquad Ans.$$

The accepted value, in Table 9.1, is 6100 ft/s. This accuracy (3%) is very good, considering the small change in density (1.2%).

P1.80 In Problem P1.24, for the given data, the air velocity at section 2 is 1180 ft/s. What is the Mach number at that section?

Solution: First convert 1180 ft/s to 360 m/s. With T_2 given as 223 K, evaluate the speed of sound at section 2, assuming, as stated, an ideal gas:

$$a_2 = \sqrt{kRT_2} = \sqrt{1.4(287\,m^2/s^2-K)(223\,K)} = 299\ m/s$$

$$\text{Then}\quad Ma_2 = \frac{V_2}{a_2} = \frac{360\,m/s}{299\,m/s} \approx \mathbf{1.20} \qquad Ans.$$

This kind of calculation will be a large part of the material in Chap. 9, Compressible Flow.

***P1.81** Use Eq. (1.39) to find and sketch the streamlines of the following flow field:

$$u = K(x^2 - y^2) \;;\; v = -2Kxy \;;\; w = 0 \;, \quad \text{where } K \text{ is a constant}$$

Hint: This is a first-order *exact* differential equation.

Solution: The asterisk * denotes this as a relatively difficult problem. From Eq. (1.39),

$$\frac{dx}{u} = \frac{dx}{K(x^2 - y^2)} = \frac{dy}{v} = \frac{dy}{-2Kxy}, \; or: \quad 2xy\,dx + (x^2 - y^2)\,dy = 0$$

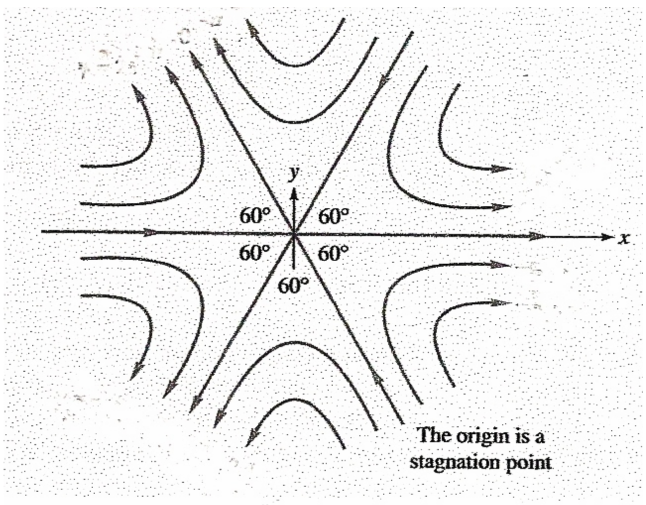
The latter form is suitable for determining 'exactness'. The equation $M\,dx + N\,dy = 0$ is exact if

$\partial M / \partial y = \partial N / \partial x$. Check this:

$\partial(2xy)/\partial y = 2x = \partial(x^2 - y^2)$, Yes, the equation is exact.

To finish, find the function F for which $\partial F / \partial x = M = 2xy$ *and* $\partial F / \partial y = x^2 - y^2$

Integrating each of these yields the final function $F = x^2y - y^3/3$ = constant, which is the equation of the streamlines for this problem. The figure shows the flow patterns for various values of the constant. It is as if the flow were trapped between six 60º walls. The arrows are determined by checking the directions of u and v in each of the six segments.



P1.82 A velocity field is given by u = V cosθ, v = V sinθ, and w = 0, where V and θ are constants. Find an expression for the streamlines of this flow.

Solution: Equation (1.44) may be used to find the streamlines:

$$\frac{dx}{u} = \frac{dy}{v} = \frac{dx}{V\cos\theta} = \frac{dy}{V\sin\theta}, \quad \text{or:} \quad \frac{dy}{dx} = \tan\theta$$

$$\text{Solution:} \quad y = (\tan\theta)\,x + constant \quad \textit{Ans.}$$

The streamlines are straight parallel lines which make an angle θ with the x axis. In other words, this velocity field represents a *uniform stream V moving upward at angle* θ.

***P1.83** A two-dimensional *unsteady* velocity field is given by u = x(1 + 2t), v = y. Find the time-varying streamlines which pass through some reference point (x_o,y_o). Sketch some.

Solution: Equation (1.44) applies with time as a parameter:

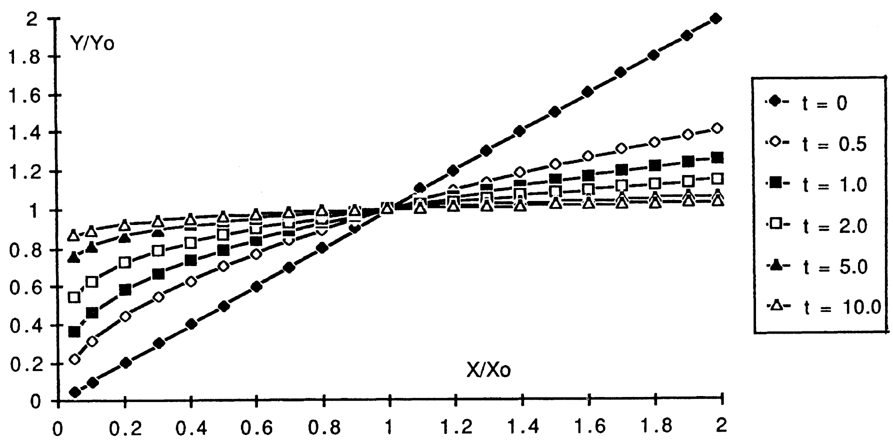
$$\frac{dx}{u} = \frac{dx}{x(1+2t)} = \frac{dy}{v} = \frac{dy}{y}, \quad \text{or:} \quad \ln(y) = \frac{1}{1+2t}\ln(x) + \text{constant}$$

$$\text{or:} \quad y = Cx^{1/(1+2t)}, \quad \text{where C is a constant}$$

In order for all streamlines to pass through y = y_o at x = x_o, the constant must be such that:

$$\boldsymbol{y = y_o(x/x_o)^{1/(1+2t)}} \quad \textit{Ans.}$$

Some streamlines are plotted on the next page and are seen to be strongly time-varying.



P1.84 In the early 1900's, the British chemist Sir Cyril Hinshelwood quipped that fluid dynamics was divided into "workers who observed things they could not explain and workers who explained things they could not observe". To what historic situation was he referring?

Solution: He was referring to the split between hydraulics engineers, who performed experiments but had no theory for what they were observing, and theoretical hydrodynamicists, who found numerous mathematical solutions, for inviscid flow, that were not verified by experiment.

P1.85-a Report to the class on the achievements of *Evangelista Torricelli*.

Solution: Torricelli's biography is taken from a goldmine of information which I did not put in the references, preferring to let the students find it themselves: C. C. Gillespie (ed.), *Dictionary of Scientific Biography*, 15 vols., Charles Scribner's Sons, New York, 1976.

Torricelli (1608–1647) was born in Faenza, Italy, to poor parents who recognized his genius and arranged through Jesuit priests to have him study mathematics, philosophy, and (later) hydraulic engineering under Benedetto Castelli. His work on dynamics of projectiles attracted the attention of Galileo himself, who took on Torricelli as an assistant in 1641. Galileo died one year later, and Torricelli was appointed in his place as "mathematician and philosopher" by Duke Ferdinando II of Tuscany. He then took up residence in Florence, where he spent his five happiest years, until his death in 1647. In 1644 he published his only known printed work, *Opera Geometrica*, which made him famous as a mathematician and geometer.

In addition to many contributions to geometry and calculus, Torricelli was the first to show that a zero-drag projectile formed a *parabolic* trajectory. His tables of trajectories for various angles and initial velocities were used by Italian artillerymen. He was an excellent machinist and constructed—and sold—the very finest telescope lenses in Italy.

Torricelli's hydraulic studies were brief but stunning, leading Ernst Mach to proclaim him the 'founder of hydrodynamics.' He deduced his theorem that the velocity of efflux from a hole in a tank was equal to $\sqrt{(2gh)}$, where h is the height of the free surface above the hole. He also showed that the efflux jet was parabolic and even commented on water-droplet breakup and the effect of air resistance. By experimenting with various liquids in closed tubes—including mercury (from mines in Tuscany)—he thereby invented the *barometer*. From barometric pressure (about 30 feet of water) he was able to explain why siphons did not work if the elevation change was too large. He also was the first to explain that winds were produced by temperature and *density differences* in the atmosphere and not by "evaporation."

P1.85-b Report to the class on the achievements of *Henri de Pitot*.

Solution: The following notes are abstracted from the *Dictionary of Scientific Biography* (see Prob. 1.85-a).

Pitot (1695–1771) was born in Aramon, France, to patrician parents. He hated to study and entered the military instead, but only for a short time. Chance reading of a textbook obtained in Grenoble led him back to academic studies of mathematics, astronomy, and engineering. In 1723 he became assistant to Réamur at the French Academy of Sciences and in 1740 became a civil engineer upon his appointment as a director of public works in Languedoc Province. He retired in 1756 and returned to Aramon until his death in 1771.

Pitot's research was apparently mediocre, described as "competent solutions to minor problems without lasting significance"—not a good recommendation for tenure nowadays! His *lasting* contribution was the invention, in 1735, of the instrument which bears his name: a glass tube bent at right angles and inserted into a moving stream with the opening facing upstream. The water level in the tube rises a distance *h* above the surface, and Pitot correctly deduced that the stream velocity ≈ $\sqrt{(2gh)}$. This is still a basic instrument in fluid mechanics.

P1.85-c Report to the class on the achievements of *Antoine Chézy*.

Solution: The following notes are from Rouse and Ince [Ref. 12].

Chézy (1718–1798) was born in Châlons-sur-Marne, France, studied engineering at the Ecole des Ponts et Chaussées and then spent his entire career working for this school, finally being appointed Director one year before his death. His chief contribution was to study the flow in open channels and rivers, resulting in a famous formula, used even today, for the average velocity:

$$V \approx \text{const}\sqrt{AS/P}$$

where A is the cross-section area, S the bottom slope, and P the wetted perimeter, i.e., the length of the bottom and sides of the cross-section. The "constant" depends primarily on the roughness of the channel bottom and sides. [See Chap. 10 for further details.]

P1.85-d Report to the class on the achievements of *Gotthilf Heinrich Ludwig Hagen*.

Solution: The following notes are from Rouse and Ince [Ref. 12].

Hagen (1884) was born in Königsberg, East Prussia, and studied there, having among his teachers the famous mathematician Bessel. He became an engineer, teacher, and writer and published a handbook on hydraulic engineering in 1841. He is best known for his study in 1839 of pipe-flow resistance, for water flow at heads of 0.7 to 40 cm, diameters of 2.5 to 6 mm, and lengths of 47 to 110 cm. The measurements indicated that the pressure drop was proportional to Q at low heads and proportional (approximately) to

Q^2 at higher heads, where "strong movements" occurred—turbulence. He also showed that Δp was approximately proportional to D^{-4}.

Later, in an 1854 paper, Hagen noted that the difference between laminar and turbulent flow was clearly visible in the efflux jet, which was either "smooth or fluctuating," and in glass tubes, where sawdust particles either "moved axially" or, at higher Q, "came into whirling motion." Thus Hagen was a true pioneer in fluid mechanics experimentation. Unfortunately, his achievements were somewhat overshadowed by the more widely publicized 1840 tube-flow studies of J. L. M. Poiseuille, the French physician.

P1.85-e Report to the class on the achievements of *Julius Weisbach*.

Solution: The following notes are abstracted from the *Dictionary of Scientific Biography* (see Prob. 1.85-a) and also from Rouse and Ince [Ref. 12].

Weisbach (1806–1871) was born near Annaberg, Germany, the 8th of nine children of working-class parents. He studied mathematics, physics, and mechanics at Göttingen and Vienna and in 1931 became instructor of mathematics at Freiberg Gymnasium. In 1835 he was promoted to full professor at the Bergakademie in Freiberg. He published 15 books and 59 papers, primarily on hydraulics. He was a skilled laboratory worker and summarized his results in *Experimental-Hydraulik* (Freiberg, 1855) and in the *Lehrbuch der Ingenieur- und Maschinen-Mechanik* (Brunswick, 1845), which was still in print 60 years later. There were 13 chapters on hydraulics in this latter treatise. Weisbach modernized the subject of fluid mechanics, and his discussions and drawings of flow patterns would be welcome in any 20th century textbook—see Rouse and Ince [23] for examples.

Weisbach was the first to write the pipe-resistance head-loss formula in modern form: $h_{f(pipe)} = f(L/D)(V^2/2g)$, where f was the dimensionless 'friction factor,' which Weisbach noted was not a constant but related to the pipe flow parameters [see Sect. 6.4]. He was also the first to derive the "weir equation" for volume flow rate Q over a dam of crest length L:

$$Q \approx \frac{2}{3}C_w(2g)^{1/2}\left[\left(H+\frac{V^2}{2g}\right)^{3/2}-\left(\frac{V^2}{2g}\right)^{3/2}\right] \approx \frac{2}{3}C_w(2g)^{1/2}H^{3/2}$$

where H is the upstream water head level above the dam crest and C_W is a dimensionless weir coefficient ≈ O(unity). [see Sect. 10.7] In 1860 Weisbach received the first Honorary Membership awarded by the German engineering society, the *Verein Deutscher Ingenieure*.

P1.85-f Report to the class on the achievements of *George Gabriel Stokes*.

Solution: The following notes are abstracted from the *Dictionary of Scientific Biography* (see Prob. 1.85-a).

Stokes (1819–1903) was born in Skreen, County Sligo, Ireland, to a clergical family associated for generations with the Church of Ireland. He attended Bristol College and Cambridge University and, upon graduation in 1841, was elected Fellow of Pembroke College, Cambridge. In 1849, he became Lucasian Professor at Cambridge, a post once held by Isaac Newton. His 60-year career was spent primarily at Cambridge and resulted in many honors: President of the Cambridge Philosophical Society (1859), secretary (1854) and president (1885) of the Royal Society of London, member of Parliament (1887–1891), knighthood (1889), the Copley Medal (1893), and Master of Pembroke College (1902). A true 'natural philosopher,' Stokes systematically explored hydrodynamics, elasticity, wave mechanics, diffraction, gravity, acoustics, heat, meteorology, and chemistry. His primary research output was from 1840–1860, for he later became tied down with administrative duties.

In hydrodynamics, Stokes has several formulas and fields named after him:

(1) The equations of motion of a linear viscous fluid: the *Navier-Stokes equations*.
(2) The motion of nonlinear deep-water surface waves: *Stokes waves*.
(3) The drag on a sphere at low Reynolds number: *Stokes' formula*, $F = 3\pi\mu VD$.
(4) Flow over immersed bodies for $Re \ll 1$: *Stokes flow*.
(5) A metric (CGS) unit of kinematic viscosity, ν: $1\ cm^2/s = 1$ *stoke*.
(6) A relation between the 1st and 2nd coefficients of viscosity: *Stokes' hypothesis*.
(7) A stream function for axisymmetric flow: *Stokes' stream function* [see Chap. 8].

Although Navier, Poisson, and Saint-Venant had made derivations of the equations of motion of a viscous fluid in the 1820's and 1830's, Stokes was quite unfamiliar with the French literature. He published a completely independent derivation in 1845 of the *Navier-Stokes equations* [see Sect. 4.3], using a 'continuum-calculus' rather than a 'molecular' viewpoint, and showed that these equations were directly analogous to the motion of elastic solids. Although not really new, Stokes' equations were notable for being the first to replace the mysterious French 'molecular coefficient' ε by the coefficient of absolute viscosity, μ.

P1.85-g Report to the class on the achievements of *Moritz Weber*.

Solution: The following notes are from Rouse and Ince [Ref. 12].

Weber (1871–1951) was professor of naval mechanics at the Polytechnic Institute of Berlin. He clarified the principles of similitude (dimensional analysis) in the form used today. It was he who named the Froude number and the Reynolds number in honor of those workers. In a 1919 paper, he developed a dimensionless surface-tension (capillarity) parameter [see Sect. 5.4] which was later named the *Weber number* in his honor.

P1.85-h Report to the class on the achievements of *Theodor von Kármán.*

Solution: The following notes are abstracted from the *Dictionary of Scientific Biography* (see Prob. 1.85-a). Another good reference is his ghost-written (by Lee Edson) autobiography, *The Wind and Beyond*, Little-Brown, Boston, 1967.

Kármán (1881–1963) was born in Budapest, Hungary, to distinguished and well-educated parents. He attended the Technical University of Budapest and in 1906 received a fellowship to Göttingen, where he worked for six years with Ludwig Prandtl, who had just developed boundary layer theory. He received a doctorate in 1912 from Göttingen and was then appointed director of aeronautics at the Polytechnic Institute of Aachen. He remained at Aachen until 1929, when he was named director of the newly formed Guggenheim Aeronautical Laboratory at the California Institute of Technology. Kármán developed CalTech into a premier research center for aeronautics. His leadership spurred the growth of the aerospace industry in southern California. He helped found the Jet Propulsion Laboratory and the Aerojet General Corporation. After World War II, Kármán founded a research arm for NATO, the Advisory Group for Aeronautical Research and Development, whose renowned educational institute in Brussels is now called the Von Kármán Center.

Kármán was uniquely skilled in integrating physics, mathematics, and fluid mechanics into a variety of phenomena. His most famous paper was written in 1912 to explain the puzzling alternating vortices shed behind cylinders in a steady-flow experiment conducted by K. Hiemenz, one of Kármán's students—these are now called *Kármán vortex streets* [see Fig. 5.2a]. Shed vortices are thought to have caused the destruction by winds of the Tacoma Narrows Bridge in 1940 in Washington State.

Kármán wrote 171 articles and 5 books and his methods had a profound influence on fluid mechanics education in the 20th century.

P1.85-i Report to the class on the achievements of *Paul Richard Heinrich Blasius.*

Solution: The following notes are from Rouse and Ince [Ref. 12].

Blasius (1883–1970) was Ludwig Prandtl's first graduate student at Göttingen. His 1908 dissertation gave the analytic solution for the laminar boundary layer on a flat plate [see Sect. 7.4]. Then, in two papers in 1911 and 1913, he gave the first demonstration that pipe-flow resistance could be nondimensionalized as a plot of friction factor versus Reynolds number—the first "Moody-type" chart. His correlation, $f \approx 0.316\, Re_d^{-1/4}$, is still is use today. He later worked on analytical solutions of boundary layers with variable pressure gradients.

P1.85-j Report to the class on the achievements of *Ludwig Prandtl*.

Solution: The following notes are from Rouse and Ince [Ref. 12].

Ludwig Prandtl (1875–1953) is described by Rouse and Ince [23] as the father of modern fluid mechanics. Born in Munich, the son of a professor, Prandtl studied engineering and received a doctorate in elasticity. But his first job as an engineer made him aware of the lack of correlation between theory and experiment in fluid mechanics. He conducted research from 1901–1904 at the Polytechnic Institute of Hanover and presented a seminal paper in 1904, outlining the new concept of "boundary layer theory." He was promptly hired as professor and director of applied mechanics at the University of Gottingen, where he remained throughout his career. He, and his dozens of famous students, started a new "engineering science" of fluid mechanics, emphasizing (1) mathematical analysis based upon by physical reasoning; (2) new experimental techniques; and (3) new and inspired flow-visualization schemes which greatly increased our understanding of flow phenomena.

In addition to boundary-layer theory, Prandtl made important contributions to (1) wing theory; (2) turbulence modeling; (3) supersonic flow; (4) dimensional analysis; and (5) instability and transition of laminar flow. He was a legendary engineering professor.

P1.85-k Report to the class on the achievements of *Osborne Reynolds*.

Solution: The following notes are from Rouse and Ince [Ref. 12].

Osborne Reynolds (1842–1912) was born in Belfast, Ireland, to a clerical family and studied mathematics at Cambridge University. In 1868 he was appointed chair of engineering at a college which is now known as the University of Manchester Institute of Science and Technology (UMIST). He wrote on wide-ranging topics—mechanics, electricity, navigation—and developed a new hydraulics laboratory at UMIST. He was the first person to demonstrate cavitation, that is, formation of vapor bubbles due to high velocity and low pressure. His most famous experiment, still performed in the undergraduate laboratory at UMIST (see Fig. 6.5 in the text) demonstrated transition of laminar pipe flow into turbulence. He also showed in this experiment that the viscosity was very important and led him to the dimensionless stability parameter $\rho VD/\mu$ now called the *Reynolds number* in his honor. Perhaps his most important paper, in 1894, extended the Navier-Stokes equations (see Eqs. 4.38 of the text) to time-averaged randomly fluctuating turbulent flow, with a result now called the *Reynolds equations* of turbulence. Reynolds also contributed to the concept of the *control volume* which forms the basis of integral analysis of flow (Chap. 3).

P1.85-l Report to the class on the achievements of *John William Strutt, Lord Rayleigh.*

Solution: The following notes are from Rouse and Ince [Ref. 12].

John William Strutt (1842–1919) was born in Essex, England, and inherited the title Lord Rayleigh. He studied at Cambridge University and was a traditional hydro-dynamicist in the spirit of Euler and Stokes. He taught at Cambridge most of his life and also served as president of the Royal Society. He is most famous for his work (and his textbook) on the theory of sound. In 1904 he won the Nobel Prize for the discovery of argon gas. He made at least five important contributions to hydrodynamics: (1) the equations of bubble dynamics in liquids, now known as *Rayleigh-Plesset theory*; (2) the theory of nonlinear surface waves; (3) the capillary (surface tension) instability of jets; (4) the “heat-transfer analogy” to laminar flow; and (5) dimensional similarity, especially related to viscosity data for argon gas and later generalized into group theory which previewed Buckingham’s Pi Theorem. He ended his career as president, in 1909, of the first British committee on aeronautics.

P1.85-m Report to the class on the achievements of *Daniel Bernoulli.*

Solution: The following notes are from Rouse and Ince [Ref. 12].

Daniel Bernoulli (1700–1782) was born in Groningen, Holland, his father, Johann, being a Dutch professor. He studied at the University of Basel, Switzerland, and taught mathematics for a few years at St. Petersburg, Russia. There he wrote, and published in 1738, his famous treatise *Hydrodynamica*, for which he is best known. This text contained numerous ingenious drawings illustrating various flow phenomena. Bernoulli used energy concepts to establish proportional relations between kinetic and potential energy, with pressure work added only in the abstract. Thus he never actually derived the famous equation now bearing his name (Eq. 3.77 of the text), later derived in 1755 by his friend Leonhard Euler. Daniel Bernoulli never married and thus never contributed additional members to his famous family of mathematicians.

P1.85-n Report to the class on the achievements of *Leonhard Euler*.

Solution: The following notes are from Rouse and Ince [Ref. 12].

Leonhard Euler (1707–1783) was born in Basel, Switzerland, and studied mathematics under Johann Bernoulli, Daniel's father. He succeeded Daniel Bernoulli as professor of mathematics at the St. Petersburg Academy, leaving there in 1741 to join the faculty of Berlin University. He lost his sight in 1766 but continued to work, aided by a prodigious memory, and produced a vast output of scientific papers, dealing with mathematics, optics, mechanics, hydrodynamics, and celestial mechanics (for which he is most famous today). His famous paper of 1755 on fluid flow derived the full inviscid equations of fluid motion (Eqs. 4.36 of the text) now called *Euler's equations*. He used a fixed coordinate system, now called the *Eulerian frame of reference*. The paper also presented, for the first time, the correct form of Bernoulli's equation (Eq. 3.77 of the text). Separately, in 1754 he produced a seminal paper on the theory of reaction turbines, leading to *Euler's turbine equation* (Eq. 11.11 of the text).

P1.86 A right circular cylinder volume υ is to be calculated from the measured base radius R and height H. If the uncertainty in R is 2% and the uncertainty in H is 3%, estimate the overall uncertainty in the calculated volume.

Solution: The formula for volume is, of course, $\upsilon = \pi R^2 H$. There are two terms to be calculated on the right-hand side of Eq. (1.43):

$$\frac{\partial \upsilon}{\partial R}\delta R = 2\pi RH\,\delta R \quad ; \quad \frac{\partial \upsilon}{\partial H}\delta H = \pi R^2\,\delta H$$

Expressed as a ratio to the volume, these become

$$\frac{\delta \upsilon}{\upsilon}\Big|_{due\,to\,R} = \frac{2\pi RH\,\delta R}{\pi R^2 H} = 2\frac{\delta R}{R} \quad ; \quad \frac{\delta \upsilon}{\upsilon}\Big|_{due\,to\,H} = \frac{\pi R^2\,\delta H}{\pi R^2 H} = \frac{\delta H}{H}$$

Note $\delta R/R$ = 2% and $\delta H/H$ = 3%. The overall uncertainty in the volume is a root-mean-square:

$$\frac{\delta \upsilon}{\upsilon} = [(2\frac{\delta R}{R})^2 + (\frac{\delta H}{H})^2]^{1/2} = [\{2(2\%)\}^2 + (3\%)^2]^{1/2} = (16+9)^{1/2} = (25)^{1/2} = \mathbf{5\%} \qquad \textit{Ans.}$$

Because it is doubled, the error in radius contributes more to the overall uncertainty.

P1.87 A dimensionless parameter, important in natural convection heat transfer of fluids, is the *Grashof number*:

$$\text{Gr} = \frac{g\ \beta\ \rho^2\ L^3\ \Delta T}{\mu^2}$$

where g is the acceleration of gravity, β is the thermal expansion coefficient, ρ the density, L a characteristic length, ΔT a temperature difference, and μ the viscosity. If the uncertainty of each of these variables is ±2 per cent, determine the overall uncertainty of the Grashof number.

Solution: This grouping of variables fits the conditions of Eq. (1.44):

$$\frac{\delta \mathrm{Gr}}{\mathrm{Gr}} = [(\frac{\delta g}{g})^2 + (\frac{\delta \beta}{\beta})^2 + (2\frac{\delta \rho}{\rho})^2 + (3\frac{\delta L}{L})^2 + (\frac{\delta \Delta T}{\Delta T}) + (2\frac{\delta \mu}{\mu})^2]^{1/2}$$
$$= [(0\%)^2 + (2\%)^2 + \{2(2\%)\}^2 + \{3(2\%)\}^2 + (2\%)^2 + \{2(2\%)\}^2]^{1/2}$$
$$= \sqrt{76} = \pm \mathbf{8.7\%} \qquad \textit{Ans.}$$

The fact that the length error is cubed contributes about two-thirds of this uncertainty.

P1.88 The device in Fig. P1.54 is called a *rotating disk viscometer* [29]. Suppose that R = 5 cm and h = 1 mm. (*a*) If the torque required to rotate the disk at 900 r/min is 0.537 N·m, what is the viscosity of the fluid? (*b*) If the uncertainty in each parameter (M, R, h, Ω) is ±1%, what is the overall uncertainty in the viscosity?

Solution: From Prob. 1.54, the analytical result for the required torque M is

$$M = \frac{\pi \mu \Omega R^4}{h}, \qquad \text{rewrite this as} \qquad \mu = \frac{M h}{\pi \Omega R^4}$$

(a) First convert Ω to radians: Ω = 900 r/min x 2π/60 = 94.2 rad/s. Then apply the formula:

$$\mu = \frac{M h}{\pi \Omega R^4} = \frac{(0.537\,N-m)(0.001m)}{\pi(94.2 rad/s)(0.05m)^4} = 0.29\frac{N-s}{m^2} = \mathbf{0.29\ \frac{kg}{m-s}} \qquad \textit{Ans.(a)}$$

This could be SAE 30W oil!

(*b*) in calculating uncertainty, the only complication is the term R^4, whose uncertainty, from Eq. (1.44), is 4 times the uncertainty in R:

$$\frac{\delta R^4}{R^4} \approx 4\frac{\delta R}{R}$$

That said, the overall uncertainty in viscosity is calculated as

$$\frac{\delta \mu}{\mu} = [(\frac{\delta M}{M})^2 + (\frac{\delta h}{h})^2 + (\frac{\delta \Omega}{\Omega})^2 + (\frac{\delta R^4}{R^4})^2]^{1/2}$$
$$= [(1\%)^2 + (1\%)^2 + (1\%)^2 + \{4(1\%)\}^2]^{1/2} = \sqrt{19} = \mathbf{4.4\%} \qquad \textit{Ans.(b)}$$

Clearly the error in R is responsible for almost all of the uncertainty in the viscosity.

P1.89 For the cone-plate viscometer of Fig. P1.56, suppose R = 6 cm and θ = 3°.
(*a*) If the torque M required to rotate the cone at 600 r/min is 0.157 N·m, what is the viscosity of the fluid?
(*b*) If the uncertainty in each parameter (M, R, θ, Ω) is ±2%, what is the overall uncertainty in the viscosity?

Solution: First convert 600 r/min to (600)(2π/60) = 62.8 rad/s. (a) From Prob. 1.5, the analytical result for the measured viscosity μ is calculated as

$$\mu = \frac{3M\sin\theta}{2\pi\Omega R^3} = \frac{3(0.157N-m)\sin(3^\circ)}{2\pi(62.8\,rad/s)(0.06m)^3} = 0.29\frac{N-s}{m^2} = \mathbf{0.29\frac{kg}{m-s}} \quad Ans.(a)$$

Once again, this could be SAE 30W oil! (*b*) The only complication in the uncertainty calculation is that the error in R^3 is three times the error in R:

$$\frac{\delta R^3}{R^3} = \frac{(\frac{\partial R^3}{\partial R})\delta R}{R^3} = \frac{3R^2\,\delta R}{R^3} = 3\frac{\delta R}{R}$$

Then the overall uncertainty is computed as

$$\frac{\delta\mu}{\mu} = [(\frac{\delta M}{M})^2 + (\frac{\delta\theta}{\theta})^2 + (\frac{\delta\Omega}{\Omega})^2 + (\frac{\delta R^3}{R^3})^2]^{1/2}$$

$$= [(2\%)^2 + (2\%)^2 + (2\%)^2 + \{3(2\%)\}^2]^{1/2} = \sqrt{48} = \mathbf{6.9\%} \quad Ans.(b)$$

The radius R, whose error is tripled, dominates the uncertainty.

P1.90 The dimensionless *drag coefficient* C_D of a sphere, to be studied in Chaps. 5 and 7, is

$$C_D = \frac{F}{(1/2)\rho V^2 (\pi/4) D^2}$$

where F is the drag force, ρ the fluid density, V the fluid velocity, and D the sphere diameter. If the uncertainties of these variables are F (±3%), ρ (±1.5%), V (±2%), and D (±1%), what is the overall uncertainty in the measured drag coefficient?

Solution: Since F and ρ occur alone, i.e. to the 1st power, their uncertainties are as stated. However, both V and D are squared, so the relevant uncertainties are doubled:

$$\frac{\delta V^2}{V^2} = 2\frac{\delta V}{V} \quad ; \quad \frac{\delta D^2}{D^2} = 2\frac{\delta D}{D}$$

Putting together the four different uncertainties in the definition of C_D, we obtain

$$\frac{\delta C_D}{C_D} = \left[\left(\frac{\delta F}{F}\right)^2 + \left(\frac{\delta \rho}{\rho}\right)^2 + \left(\frac{\delta V^2}{V^2}\right)^2 + \left(\frac{\delta D^2}{D^2}\right)^2\right]^{1/2}$$

$$= [(3\%)^2 + (1.5\%)^2 + \{2(2\%)\}^2 + \{2(1\%)\}^2] = \sqrt{31.25} = \mathbf{5.6\%} \quad \textit{Ans.}$$

The largest contribution comes from the uncertainty in velocity.

FUNDAMENTALS OF ENGINEERING EXAM PROBLEMS: Answers

FE-1.1 The absolute viscosity μ of a fluid is primarily a function of
(a) density **(b) temperature** (c) pressure (d) velocity (e) surface tension

FE-1.2 Carbon dioxide, at 20°C and 1 atm, is compressed isentropically to 4 atm. Assume CO_2 is an ideal gas. The final temperature would be
(*a*) **130°C** , (*b*) 162°C , (*c*) 171°C , (*d*) 237°C , (*e*) 313°C

FE-1.3 Helium has a molecular weight of 4.003. What is the weight of 2 cubic meters of helium at 1 atmosphere and 20°C?
(a) 3.3 N (b) 6.5 N (c) 11.8 N (d) 23.5 N (e) 94.2 N

FE-1.4 An oil has a kinematic viscosity of 1.25E–4 m^2/s and a specific gravity of 0.80. What is its dynamic (absolute) viscosity in $kg/(m \cdot s)$?
(a) 0.08 **(b) 0.10** (c) 0.125 (d) 1.0 (e) 1.25

FE-1.5 Consider a soap bubble of diameter 3 mm. If the surface tension coefficient is 0.072 N/m and external pressure is 0 Pa gage, what is the bubble's internal gage pressure?
(a) −24 Pa (b) +48 Pa (c) +96 Pa **(d) +192 Pa** (e) −192 Pa

FE-1.6 The only possible dimensionless group which combines velocity V, body size L, fluid density ρ, and surface tension coefficient σ is:
(a) $L\rho\sigma/V$ (b) $\rho VL^2/\sigma$ (c) $\rho\sigma V^2/L$ (d) $\sigma LV^2/\rho$ **(e) $\rho LV^2/\sigma$**

FE-1.7 Two parallel plates, one moving at 4 m/s and the other fixed, are separated by a 5-mm-thick layer of oil of specific gravity 0.80 and kinematic viscosity 1.25E−4 m^2/s. What is the average shear stress in the oil?
(a) 80 Pa (b) 100 Pa (c) 125 Pa (d) 160 Pa (e) 200 Pa

FE-1.8 Carbon dioxide has a specific heat ratio of 1.30 and a gas constant of 189 J/(kg·°C). If its temperature rises from 20°C to 45°C, what is its internal energy rise?
(a) 12.6 kJ/kg **(b) 15.8 kJ/kg** (c) 17.6 kJ/kg (d) 20.5 kJ/kg (e) 25.1 kJ/kg

FE-1.9 A certain water flow at 20°C has a critical cavitation number, where bubbles form, Ca ≈ 0.25, where $Ca = 2(p_a - p_{vap})/(\rho V^2)$. If p_a = 1 atm and the vapor pressure is 0.34 psia, for what water velocity will bubbles form?
(a) 12 mi/hr (b) 28 mi/hr (c) 36 mi/hr (d) 55 mi/hr **(e) 63 mi/hr**

FE-1.10 Example 1.10 gave an analysis which predicted that the viscous moment on a rotating disk was $M = \pi\mu\Omega R^4/(2h)$. If the uncertainty of each of the four variables (μ, Ω, R, h) is 1.0 %, what is the estimated overall uncertainty of the moment M?

(*a*) 4.0 % , (*b*) **4.4 %** , (*c*) 5.0 % , (*d*) 6.0 % , (*e*) 7.0 %

COMPREHENSIVE PROBLEMS

C1.1 Sometimes equations can be developed and practical problems solved by knowing nothing more than the dimensions of the key parameters. For example, consider the heat loss through a window in a building. Window efficiency is rated in terms of "R value," which has units of $ft^2\cdot hr\cdot °F/Btu$. A certain manufacturer offers a double-pane window with R = 2.5 and also a triple-pane window with R = 3.4. Both windows are 3 ft by 5 ft. On a given winter day, the temperature difference between inside and outside is 45°F. (a) Develop and equation for window heat loss Q, in time period Δt, as a function of window area A, R value, and temperature difference ΔT. How much heat is lost through the above (a) double-pane window, or (b) triple-pane window, in 24 hours? (c) Suppose the building is heated with propane gas, at $3.25 per gallon, burning at 80% efficiency. Propane has 90,000 Btu of available energy per gallon. In a 24-hour period, how much money would a homeowner save, per window, by installing a triple-pane rather than a double-pane window? (d) Finally, suppose the homeowner buys 20 such triple-pane windows for the house. A typical winter equals about 120 heating days at ΔT = 45°F. Each triple-pane window costs $85 more than the double-pane window. Ignoring interest and inflation, how many years will it take the homeowner to make up the additional cost of the triple-pane windows from heating bill savings?

Solution: (a) The function Q = fcn(Δt, R, A, ΔT) must have units of Btu. The only combination of units which accomplishes this is:

$$Q = \frac{\Delta \mathbf{t} \Delta T A}{R} \quad Ans. \qquad Thus\ \ Q_{lost} = \frac{(24\ hr)(45°F)(3\ ft\cdot 5\ ft)}{2.5\ ft^2\cdot hr\cdot °F/Btu} = \mathbf{6480\ Btu} \quad Ans.\text{(a)}$$

(b) Triple-pane window: use R = 3.4 instead of 2.5 to obtain $Q_{3\text{-pane}}$ = **4760 Btu** *Ans.* (b)

(c) The savings, using propane, for one triple-pane window for one 24-hour period is:

$$\Delta Cost = \frac{\$3.25/gal}{90000\,Btu/gal}(6480 - 4760\,Btu)\frac{1}{0.80_{efficiency}} = \$0.078 = \mathbf{7.8\ cents} \quad Ans.(c)$$

(d) Extrapolate to 20 windows, 120 cold days per year, and $85 extra cost per window:

$$Pay-back\ time = \frac{\$85/window}{(0.078\,\$/window/day)(120\,days/year)} = \mathbf{9\ years} \quad Ans.(d)$$

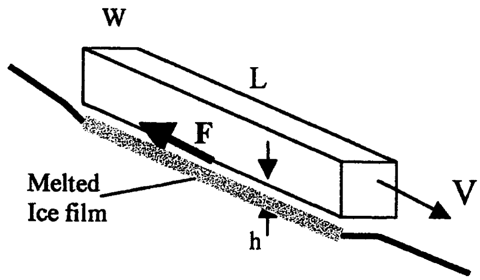
Not a very good investment. We are using '$' and 'windows' as "units" in our equations!

C1.2 When a person ice-skates, the ice surface actually melts beneath the blades, so that he or she skates on a thin film of water between the blade and the ice. (a) Find an expression for total friction force F on the bottom of the blade as a function of skater velocity V, blade length L, water film thickness h, water viscosity μ, and blade width W. (b) Suppose a skater of mass *m*, moving at constant speed V_o, suddenly stands stiffly with skates pointed directly forward and allows herself to coast to a stop. Neglecting air resistance, how far will she travel (on *two* blades) before she stops? Give the answer X as a function of (V_o, *m*, L, h, μ, W). (c) Compute X for the case V_o = 4 m/s, m = 100 kg, L = 30 cm, W = 5 mm, and h = 0.1 mm. Do you think our assumption of negligible air resistance was a good one?

Solution: (a) The skate bottom and the melted ice are like two parallel plates:

$$\tau = \mu\frac{V}{h}, \quad F = \tau A = \frac{\mu VLW}{h} \quad \textit{Ans. (a)}$$

(b) Use **F** = m**a** to find the stopping distance:



$$\Sigma F_x = -F = -\frac{2\mu VLW}{h} = ma_x = m\frac{dV}{dt}$$

(*the* '2' *is for two blades*)

Separate and integrate once to find the velocity, once again to find the distance traveled:

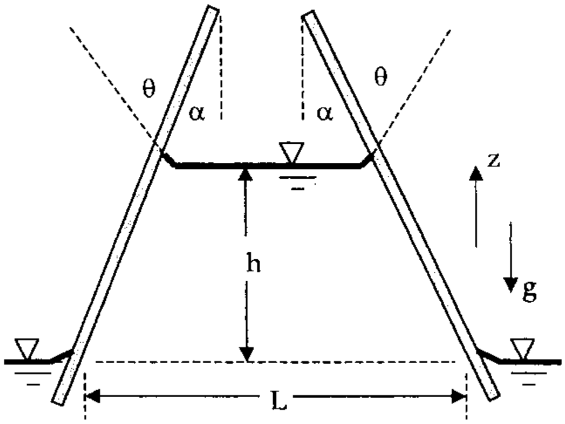
$$\int \frac{dV}{V} = -\int \frac{2\mu LW}{mh}dt, \quad or: \quad V = V_o e^{\frac{-2\mu LW}{mh}t}, \quad X = \int_0^\infty V\,dt = \frac{V_o mh}{2\mu LW} \quad \textit{Ans. (b)}$$

(c) Apply our specific numerical values to a 100-kg (!) person:

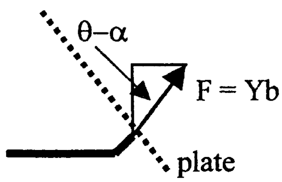
$$X = \frac{(4.0\ m/s)(100\ kg)(0.0001\ m)}{2(1.788E\text{–}3\ kg/m\cdot s)(0.3\ m)(0.005\ m)} = \mathbf{7460\ m\,(!)} \quad \textit{Ans. (c)}$$

We could coast to the next town on ice skates! It appears that our assumption of negligible air drag was grossly incorrect.

C1.3 Two thin flat plates are tilted at an angle α and placed in a tank of known surface tension Y and contact angle θ, as shown. At the free surface of the liquid in the tank, the two plates are a distance L apart, and of width b into the paper. (a) What is the total z-directed force, due to surface tension, acting on the liquid column between plates? (b) If the liquid density is ρ, find an expression for Y in terms of the other variables.



Solution: (a) Considering the right side of the liquid column, the surface tension acts tangent to the local surface, that is, along the dashed line at right. This force has magnitude F = Yb, as shown. Its vertical component is $F\cos(\theta-\alpha)$, as shown. There are two plates. Therefore, the total z-directed force on the liquid column is

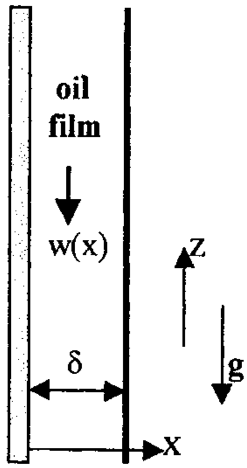


$$\mathbf{F_{vertical} = 2Yb\cos(\theta-\alpha)}\quad \textit{Ans.}\text{ (a)}$$

(b) The vertical force in (a) above holds up the entire weight of the liquid column between plates, which is $W = \rho g\{bh(L - h\tan\alpha)\}$. Set W equal to F and solve for

$$\mathbf{Y = [\rho g b h(L - h\tan\alpha)]/[2\cos(\theta-\alpha)]}\quad \textit{Ans.}\text{ (b)}$$

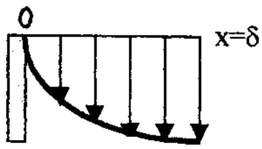
C1.4 Oil of viscosity μ and density ρ drains steadily down the side of a tall, wide vertical plate, as shown. The film is fully developed, that is, its thickness δ and velocity profile w(x) are independent of distance *z* down the plate. Assume that the atmosphere offers no shear resistance to the film surface.



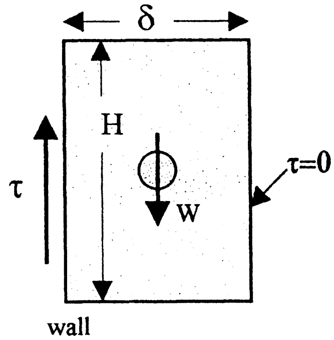
(a) Sketch the approximate shape of the velocity profile w(x), keeping in mind the boundary conditions.

(b) Suppose film thickness d is measured, along with the slope of the velocity profile at the wall, $(dw/dx)_{wall}$, with a laser-Doppler anemometer (Chap. 6). Find an expression for μ as a function of ρ, δ, $(dw/dx)_{wall}$, and g. Note that both w and $(dw/dx)_{wall}$ will be negative as shown.

Solution: (a) The velocity profile must be such that there is no slip (w = 0) at the wall and no shear (dw/dx = 0) at the film surface. This is shown at right. *Ans*. (a)



(b) Consider a freebody of any vertical length H of film, as at right. Since there is no acceleration (fully developed film), the weight of the film must exactly balance the shear force on the wall:



$$W = \rho g(H\delta b) = \tau_{wall}(Hb), \quad \tau_{wall} = -\mu\frac{dw}{dx}\Big|_{wall}$$

Solve this equality for the fluid viscosity:

$$\boldsymbol{\mu = \frac{-\rho g\delta}{(dw/dx)_{wall}}} \quad \textit{Ans}. \text{ (b)}$$

C1.5 Viscosity can be measured by flow through a thin-bore or *capillary* tube if the flow rate is low. For length L, (small) diameter $D \ll L$, pressure drop Δp, and (low) volume flow rate Q, the formula for viscosity is $\mu = D^4\Delta p/(CLQ)$, where C is a constant. (a) Verify that C is dimensionless. The following data are for water flowing through a 2-mm-diameter tube which is 1 meter long. The pressure drop is held constant at Δp = 5 kPa.

T, °C:	10.0	40.0	70.0
Q, L/min:	0.091	0.179	0.292

(b) Using proper SI units, determine an average value of C by accounting for the variation with temperature of the viscosity of water.

Solution: (a) Check the dimensions of the formula and solve for $\{C\}$:

$$\{\mu\} = \left\{\frac{M}{LT}\right\} = \left\{\frac{D^4\Delta p}{CLQ}\right\} = \left\{\frac{L^4(ML^{-1}T^{-2})}{\{C\}(L)(L^3/T)}\right\} = \left\{\frac{M}{LT\{C\}}\right\},$$

$$\textit{therefore}\ \ \mathbf{\{C\} = \{1\}}\quad \textbf{\textit{Dimensionless}}\quad \textit{Ans.}\ \text{(a)}$$

(b) Use the given data, with values of μ_{water} from Table A.1, to evaluate C, with L = 1 m, D = 0.002 m, and Δp = 5000 Pa. Convert the flow rate from L/min to m^3/s.

T, °C:	10.0	40.0	70.0
Q, m^3/s:	1.52E–6	2.98E–6	4.87E–6
μ_{water}, kg/m-s:	1.307E–3	0.657E–3	0.405E–3
$\boldsymbol{C = D^4\Delta p/(\mu LQ)}$:	40.3	40.9	40.6

The estimated value of $\boldsymbol{C = 40.6 \pm 0.3}$. The theoretical value (Chap. 4) is C = $128/\pi$ = 40.74.

C1.6 The *rotating-cylinder viscometer* in Fig. C1.6 shears the fluid in a narrow clearance, Δr, as shown. Assume a linear velocity distribution in the gaps. If the driving torque M is measured, find an expression for μ by (a) neglecting, and (b) including the bottom friction.

Solution: (a) The fluid in the annular region has the same shear stress analysis as Prob. 1.49:

$$M = \int R\,dF = \int (R)(\tau)\,dA \int_0^{2\pi} R\left(\mu\frac{\Omega R}{\Delta R}\right) RL\ d\theta = 2\pi\mu\frac{\Omega R^3 L}{\Delta R},$$

$$\textit{or:}\quad \boldsymbol{\mu = \frac{M\Delta R}{2\pi\Omega R^3 L}}\quad \textit{Ans.}\ \text{(a)}$$

(b) Now add in the moment of the (variable) shear stresses on the bottom of the cylinder:

$$M_{bottom} = \int r\tau\, dA = \int_0^R r\left(\mu \frac{\Omega r}{\Delta R}\right) 2\pi r\, dr$$

$$= \frac{2\pi\Omega\mu}{\Delta R}\int_0^R r^3\, dr = \frac{2\pi\Omega\mu R^4}{4\Delta R}$$

$$Thus\ \ M_{total} = \frac{2\pi\Omega\mu R^3 L}{\Delta R} + \frac{2\pi\Omega\mu R^4}{4\Delta R}$$

$$Solve\ for\ \ \mu = \frac{M\Delta R}{2\pi\Omega R^3 (L + R/4)} \quad Ans.\ (b)$$

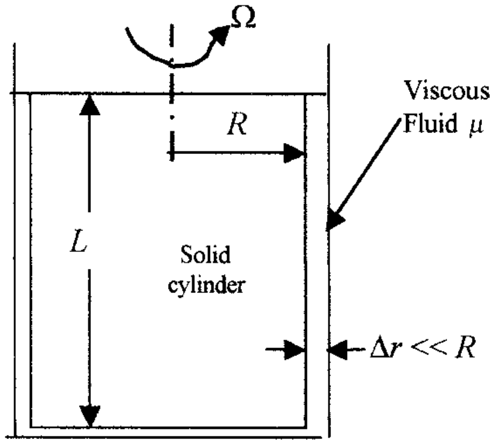
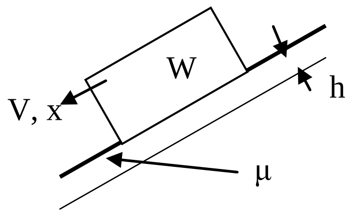


Fig. C1.6

C1.7 Make an analytical study of the transient behavior of the sliding block in Prob. 1.45. (*a*) Solve for $V(t)$ if the block starts from rest, $V = 0$ at $t = 0$. (*b*) Calculate the time t_1 when the block has reached 98% of its terminal velocity.



Solution: Let x go down the slope, and write Newton's law

$$\sum F_x = W\sin\theta - \tau A = \frac{W}{g}\frac{dV}{dt}, \text{ where } \tau = \mu\frac{V}{h}$$

Rearrange: $\frac{dV}{dt} = g\sin\theta - KV$, where $K = \frac{\mu A g}{hW}$. This is a first-order linear differential equation, with the solution $V = \frac{g\sin\theta}{K} + Ce^{-Kt}$, $C = constant$

If $V = 0$ at $t = 0$, we find that $C = -\,g\sin\theta/K$. Introduce K for the final solution:

$$V = \left(\frac{hW\sin\theta}{\mu A}\right)[1 - \exp\{-\mu Ag/(hW)\}t] \qquad Ans.(a)$$

As $t \to \infty$, $V = V_{terminal} = (hW\sin\theta/\mu A)$ as in Prob. P1.45. We are 98% there if $\exp\{-(\mu Ag/hW)\,t\} = 0.02$, or $(\mu Ag/hW)\,t_1 = -\ln(0.02) \approx 3.91$. Thus

$$t_{1-98\%} \approx 3.91\frac{hW}{\mu A g} \qquad Ans.(b)$$

C1.8 A mechanical device, which uses the rotating cylinder of Fig. C1.6, is the *Stormer viscometer* [Ref. 29 of Chap. 1]. Instead of being driven at constant Ω, a cord is wrapped around the shaft and attached to a falling weight *W*. The time *t* to turn the shaft a given number of revolutions (usually 5) is measured and correlated with viscosity. The Stormer formula is

$$t = A\mu/(W - B)$$

where *A* and *B* are constants which are determined by calibrating the device with a known fluid. Here are calibration data for a Stormer viscometer tested in glycerol, using a weight of 50 N:

μ, kg/m·s:	0.23	0.34	0.57	0.84	1.15
t, sec:	15	23	38	56	77

(a) Find reasonable values of *A* and *B* to fit this calibration data. [*Hint*: The data are not very sensitive to the value of *B*.] (b) A more viscous fluid is tested with a 100-N weight and the measured time is 44 s. Estimate the viscosity of this fluid.

Solution: (a) The data fit well, with a standard deviation of about 0.17 s in the value of *t*, to the values

$$A \approx \mathbf{3000} \quad \text{and} \quad B \approx \mathbf{3.5} \quad \textit{Ans.}\text{ (a)}$$

(b) With a new fluid and a new weight, the values of A and B should nevertheless be the same:

$$t = 44\ s \approx \frac{A\mu}{W-B} = \frac{3000\mu}{100\ N - 3.5}, \quad \textit{solve for } \mu_{new\ fluid} \approx \mathbf{1.42}\ \frac{kg}{m\cdot s} \quad \textit{Ans.}\text{ (b)}$$

C1.9 The lever in Fig. P1.101 has a weight W at one end and is tied to a cylinder at the other end. The cylinder has negligible weight and buoyancy and slides upward through a film of heavy oil of viscosity μ. (*a*) If there is no acceleration (uniform lever rotation) derive a formula for the rate of fall V_2 of the weight. Neglect the lever weight. Assume a linear velocity profile in the oil film.

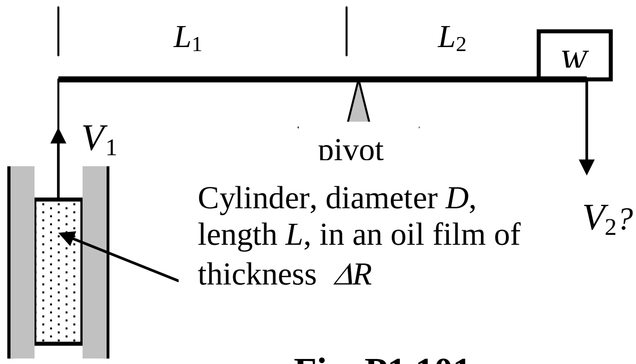


Fig. P1.101

(*b*) Estimate the fall velocity of the weight if W = 20 N, L_1 = 75 cm, L_2 = 50 cm, D = 100 cm, L = 22 cm, ΔR = 1 mm, and the oil is glycerin at 20°C.

Solution: (a) If the motion is uniform, no acceleration, then the moments balance about the pivot:

$$\sum M_{pivot} = 0 = WL_2 - F_1L_1 \ , \quad \text{where} \quad F_1 = \tau_w A_w = (\mu \frac{V_1}{\Delta R})(\pi DL)$$

Since the lever is rigid, the endpoint velocities vary according to their lengths from the pivot:

$$V_1 = \Omega L_1 \ ; \quad V_2 = \Omega L_2 \qquad \therefore \quad V_1/L_1 = V_2/L_2$$

Combine these two relations to obtain the desired solution for a highly viscous fluid:

$$V_2 = V_1(L_2/L_1) = \frac{W\,\Delta R}{\mu \pi DL}(L_2/L_1)^2 \qquad \textit{Ans.(a)}$$

(*b*) For glycerin at 20°C, from Table A.3, μ = 1.49 kg/m-s. The formula above yields

$$V_2 = \frac{W\,\Delta R}{\mu \pi DL}(\frac{L_2}{L_1})^2 = \frac{(20\,N)(0.001m)}{(1.49N-s/m^2)\pi(0.1m)(0.22\,m)}(\frac{75\,cm}{50\,cm})^2 \approx \mathbf{0.44\ \frac{m}{s}} \quad \textit{Ans.(b)}$$

C1.10 A popular gravity-driven instrument is the Cannon-Ubbelohde viscometer, shown in Fig. C1.10. The test liquid is drawn up above the bulb on the right side and allowed to drain by gravity through the capillary tube below the bulb. The time *t* for the meniscus to pass from upper to lower timing marks is recorded. The kinematic viscosity is computed by the simple formula ν = C*t*, where C is a calibration constant. For ν in the range of 100-500 mm^2/s, the recommended constant is C = 0.50 mm^2/s^2, with an accuracy less than 0.5%.

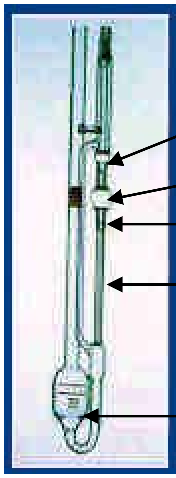


Fig. C1.10

The Cannon-Ubbelohde viscometer.

(*a*) What liquids from Table A.3 are in this viscosity range? (*b*) Is the calibration formula dimensionally consistent? (*c*) What system properties might the constant C depend upon? (*d*) What problem in this chapter hints at a formula for estimating the viscosity?

Solution: (*a*) Very hard to tell, because values of ν are not listed – sorry, I'll add these values if I remember. It turns out that only *three* of these 17 liquids are in the 100-500 mm^2/s range: SAE 10W, 10W30, and 30W oils, at 120, 194, and 326 mm^2/s respectively.

(*b*) No, the formula is dimensionally inconsistent because C has units; thus C = 0.5 is appropriate *only* for a mm^2/s result.

(*c*) Hidden in C are the length and diameter of the capillary tube and the acceleration of gravity - see Eq. (6.12) later.

(*d*) Problem P1.58 gives a formula for flow through a capillary tube. If the flow is vertical and gravity driven, $\Delta p = \rho g L$ and $Q = (\pi/4)d^2 V$, leaving a new formula as follows: $\nu \approx gd^2/(32V)$.

C1.11 Mott [Ref. 49, p. 38] discusses a simple falling-ball viscometer, which we can analyze later in Chapter 7. A small ball of diameter *D* and density ρ_b falls though a tube of test liquid.
The fall velocity *V* is calculated by the time to fall a measured distance. The formula for calculating the viscosity of the fluid is

$$\mu = \frac{(\rho_b - \rho)\, g\, D^2}{18\, V}$$

This result is limited by the requirement that the Reynolds number ($\rho VD/\mu$) be less than 1.0. Suppose a steel ball (SG = 7.87) of diameter 2.2 mm falls in SAE 25W oil (SG = 0.88) at 20°C. The measured fall velocity is 8.4 cm/s. (*a*) What is the viscosity of the oil, in kg/m-s? (*b*) Is the Reynolds number small enough for a valid estimate?

Solution: Relating SG to water, Eq. (1.7), the steel density is 7.87(1000) = 7870 kg/m^3 and the oil density is 0.88(1000) = 880 kg/m^3. Using SI units, the formula predicts

$$\mu_{oil} = \frac{(7870 - 880\, kg/m^3)(9.81\, m/s^2)(0.0022\, m)^2}{18(0.084\, m/s)} \approx \mathbf{0.22\ \frac{kg}{m-s}} \quad \textit{Ans.}$$

$$\text{Check} \quad \text{Re} = \frac{\rho VD}{\mu} = \frac{(880)(0.084)(0.0022)}{0.22} = 0.74 < 1.0 \quad \textit{OK}$$

As mentioned, we shall analyze this falling sphere problem in Chapter 7.

C1.12 A solid aluminum disk (SG = 2.7) is 2 inches in diameter and 3/16 inch thick. It slides steadily down a 14° incline that is coated with a castor oil (SG = 0.96) film one hundredth of an inch thick. The steady slide velocity is 2 cm/s. Using Figure A.1 and a linear oil velocity profile assumption, estimate the *temperature* of the castor oil.

Solution: This problem reviews complicated units, volume and weight, shear stress, and viscosity. It fits the sketch in Fig. P1.45. The writer converts to SI units.

$$W = \rho_{alum}\, g\, A h = (2700\, kg/m^3)(9.81\, m/s^2)\{\pi[(1/12)(0.3048)]^2\}(3/16/12)(0.3048) = 0.256\, N$$

The weight component along the incline balances the shear stress, in the castor oil, times the bottom flat area of the disk.

$$W \sin\theta = \mu \frac{V}{h} A \ , \ \text{or}: \ (0.256 N)\sin(14^\circ) = \mu[\frac{0.02\, m/s}{(1/100/12)(0.3048)\, m}]\pi[(1/12)(0.3048)m]^2$$

$$\text{Solve for} \quad \mu_{oil} \approx 0.39\, kg/(m-s)$$

Looking on Fig. A.1 for castor oil, this viscosity corresponds approximately to **30°C** *Ans*. The specific gravity of the castor oil was a red herring and is not needed. Note also that, since *W* is proportional to disk bottom area *A*, that area cancels out and is not needed.
